DECONVOLUTION BY COMBINED PREDICTIVE AND HOMOMORPHIC FILTERING

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CERTIFICATE

Certified that the work entitled " **DECONVOLUTION BY COMBINED PREDICTIVE AND HOMOMORPHIC FILTERING** " has been carried out under my supervision and has not been submitted elsewhere for a degree.

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CONTENTS

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION TO DECONVOLUTION :

The output of a linear system can be represented by the convolution of the input and the system impulse response. The system impulse response itself can, in many cases, be represented as a convolution of impulse response functions of many subsystems. Deconvolution is the process of removal of any or all of the convolutional factors.

Consider a linear system with impulse response , h(n). The output of the system to an input x(n) is given by

$$y(n)=x(n)*h(n)$$

There are many applications in which given y(n) and either x(n) or h(n) , we wish to determine the unknown signal. In deconvolution process , we wish to recover either h(n) or x(n) from the given information.

Deconvolution plays an important role in identification of physical systems. We excite the system with a known input , then measure the output and obtain h(n) from a knowledge of x(n) and y(n). Let q(n) be a function such that

$$x(n)*q(n) = \delta(n) \qquad (1.1)$$

where $\delta(n)$ is the unit impulse sequence. Then convolving y(n) with q(n) we get

$$y(n)*q(n)=h(n)*x(n)*q(n)=h(n)*\delta(n)=h(n)$$

The function q(n) is called the deconvolution filter , since it

"deconvolves" the input $x(n)$ from the output $y(n)$

Another variation of the deconvolution problem is to find a linear system with impulse response $g(n)$ such that

$$y(n) * g(n) = x(n)$$

which implies that

$$h(n) * g(n) = \delta(n) \tag{1.2}$$

The system $g(n)$ is known as the "inverse" of the system $h(n)$. The inverse systems find applications in situations where it is necessary to compensate for the (undesirable) effects of a linear system. This is commonly referred to as "channel equalization".

The problem of deconvolution is concerned with the development of fast and stable algorithms to implement deconvolution / inverse filters.

Deconvolution finds applications in diverse areas. Deconvolution of speech can provide the useful information of the vocal tract. In seismic signal processing , the deconvolution of a seismogram can be used to estimate the characteristics of the earth's subsurface. In passive sonar signal processing, deconvolution of the received signal can be used to estimate the target's radiated signal.

1.2 METHODS OF DECONVOLUTION :

There are a number of deconvolution schemes that are currently being used. Each method is based on certain models and assumptions. So the success of a deconvolution method depends upon how well the assumptions made by it are met in the application. There are three popular methods, namely, Predictive Deconvolution, Homomorphic Deconvolution and Deconvolution by Kalman filtering, which are briefly outlined below .

1.2.1 PREDICTIVE DECONVOLUTION [1], [9], [10], [15], [17], [21] :

This makes use of the following statistical model. The system is characterized by a random impulse response h(n) given by ,

$$h(n) = \sum_i a_i \, \delta(n - m_i) \tag{1.3}$$

where m_i is a randomly occuring point in time that can take on discrete values {0, 1, ...} ; and the a_i's are uncorrelated.

A minimum phase signal x(n) is assumed to be the input to the above system. Under the uncorrelated random feature of the system impulse response, the autocorrelation, ϕ_{yy} , of the output y(n) is given by ,

$$\phi_{yy}(n) = \phi_{xx}(n) * \phi_{hh}(n)$$

$$= \phi_{xx}(n) * K.\delta(0) \quad , \quad K \text{ is a constant.}$$

$$= K.\phi_{xx}(n) \tag{1.4}$$

Thus the autocorrelation, ϕ_{yy} , of the output y(n) is equal to the autocorrelation $\phi_{xx}(n)$ of x(n) multiplied by a constant. This scaling has no effect on deconvolution procedure and hence can be ignored. The ϕ_{yy} can be computed directly from the output y(n), and thus ϕ_{xx} can be obtained. The autocorrelation of x(n) is sufficient to determine the deconvolution filter for x(n) , viz., the prediction error operator with prediction distance equal to unity. This prediction-error operator, when convolved with x(n), produces a spike at n=0, and hence is called the deconvolution-operator. Then the system impulse response h(n) can be computed by convolving this prediction error operator with the output y(n). The prediction filter with prediction distance equal to unity is

called Wiener filter.

1.2.2 HOMOMORPHIC DECONVOLUTION [1], [3], [5], [13], [23] :

Homomorphic systems are a class of nonlinear systems which satisfy a generalized principle of superposition. Such systems are particularly useful in separating the signals combined through convolution. Consider the transformation defined by,

$$y = T\,(x).$$

If T is a linear system, it satisfies the superposition principle defined by ,

$$T\,(\,ax_1 + bx_2\,) \;=\; a\,T(x_1) \;+\; b\,T(x_2) \tag{1.5}$$

In Homomorphic signal processing, a system D, referred to as the characteristic system, transforms a convolutional space to an additive space. It is defined by the relation,

$$D\,(\,ax_1 * bx_2\,) \;=\; a\,D(x_1) \;+\; b\,D(x_2). \tag{1.6}$$

The system, D^{-1}, the inverse of D performs the transformation from an additive space back to the convolutional space. If in the additive space, the contribution due to the two convolutional factors are well separated , one of the components can be removed by low-pass or high-pass filtering in the additive space (or cepstral domain).

1.2.3 DECONVOLUTION BY KALMAN FILTERING [1] :

The deconvolution problem can be solved either by inverse filtering or calculating the prediction error. Both the methods are well known in the theory of Wiener filters. If, however, the generating process of the signal is known and can be described by a set of differential equations,

then Kalman filter can also be used to solve the deconvolution problem. The modeling of the linear system in Kalman's formulation is based on a state space representation of linear time varying systems via a first order vector *differential equation* instead of a time-varying *impulse response,* as in the case of Wiener filter. This formalism results in specification of optimum estimates as the solution to a differential equation whose coefficients are determined by the statistics of the processes involved.

Kalman filtering approach to deconvolution is applicable to time-varying or time-invarient signal as well as to nonstationary or stationary noise processes. One interesting feature of this approach is the reversal of the intuitive role of the input and the system for the same model as assumed by Predictive Deconvolution. In this case the signal x(n) is considered as the impulse response of the system and the random sequence, h(n), is considered as the input and is modeled as noise.

1.2.4 OTHER METHODS :

Besides the three techniques discussed above, there are many other methods used for deconvolution. The scheme called, *L_1 norm method* of deconvolution is suggested by Claerbout and Muir [7]. This method is said to have robustness characteristics. The *Minimum Entropy method* of deconvolution suggested by Wiggins [25] is similar to Predictive Deconvolution in the sense that operators are determined directly from the data, but Predictive Deconvolution seeks to maximize entropy as opposed to the former method, which minimizes the entropy. In the *Deterministic Deconvolution method* [1] , the input to the system is given

and is deconvolved from the output of the system. A time-varying deconvolution method has been developed which is based upon adaptive linear filtering techniques. In this *Adaptive Deconvolution* scheme [1], filter coefficients are designed for each sample of the input trace using an adaptive algorithm. The *Dynamic Predictive Deconvolution method* [18] is used for the system which is modeled as a lattice filter.

1.3 RELATIVE MERITS OF THE DECONVOLUTION METHODS:

The Predictive Deconvolution is very effective when the signal to be deconvolved is minimum phase. If the signal is nonminimum phase, then Homomorphic filtering or Kalman filtering can be applied. Further, Homomorphic Deconvolution does not require one of the convolutional factors to be random in nature, whereas for Predictive Deconvolution it is an essential requirement. However, the success of Homomorphic Deconvolution depends upon the degree of separability of the convolutional factors in the cepstral domain. Kalman filtering can be used in time-varying cases.

1.4 APPLICATIONS OF DECONVOLUTION :

1.4.1 SEISMIC SIGNAL PROCESSING :

Deconvolution is applied in seismic signal processing to estimate the physical properties of the earth's subsurface, from which the presence of oil and natural gas in the earth's subsurface can be predicted. A major goal in the seismic deconvolution is to remove all the reverberation effects to retrieve the earth's reflectivity function.

1.4.2 SPEECH PROCESSING :

Deconvolution is used in speech analysis, e.g. in an automatic speech recognition system, we begin with the speech waveform and the desired result is the recognition of the speech. Other applications are speaker identification and speaker verification. Another application is secure voice transmission and data rate compression, which involves speech analysis followed by speech synthesis. If the speech is transmitted in PCM form, the data rate required of the order of 90000 bits per second. Through the use of speech analysis using deconvolution, followed by appropriate coding, transmission and resynthesis at the receiver this can be reduced by a factor between 10 and 50.

1.4.3 SONAR SIGNAL PROCESSING :

In passive sonar operations, a target radiates an acoustic signal into an accoustic medium with surface and bottom boundaries. Generally, the boundaries of this medium produce multipath interference. Consequently, a passive receiver receives a distorted version of the radiated signal. Deconvolution can be used to suppress the effect of multipath interference in the received signal.

1.4.4 OTHER APPLICATIONS :

Deconvolution has a number of other applications. Examples include the probing of dielectric materials by electromagnetic waves, the study of optical parameters of thin films, the probing of tissues by ultrasound and the design of broadband termination of transmission lines.

1.5 MODELING IN SPEECH AND SEISMIC SIGNAL PROCESSING

Every deconvolution scheme is based on a model of the generating process. A model is a representation of a process in which the details that appear unessential for the intended use are omitted. In speech analysis, the speech waveform is modeled as the response of a linear time-varying system, the vocal tract, with the appropriate excitation. In seismic signal processing, the seismogram is modeled as the response of a linear time-invarient system, the earth, to a seismic wavelet as the input. So in this chapter, the modeling of the vocal tract and the earth as a " lattice filter " is considered.

1. 5. 1 MODELING OF SPEECH WAVEFORM :

Speech is produced by excitation of the vocal tract by *glottal pulses* [16]. If the vocal tract shape is fixed, it is modeled as a linear time-invariant system and the output is the convolution of the excitation and the vocal tract impulse response. Different sounds are produced by changing the shape of the vocal tract. If the vocal tract shape changes slowly, for short periods of time (of the order of 20 - 30 milliseconds), it may be assumed to maintain a fixed configuration.

In speech, the vocal tract is modeled as an acoustic tube of varying cross sectional area. It is considered to to be a set of interconnected acoustic tube sections of equal length and of different cross sectional area (**Fig. 1.1**). It is generally assumed that sound propagation through each section can be treated as a plane wave, and the internal losses can be ignored. The acoustic impedance of sound

wave varies inversely with the tube area, i.e. $Z = (\rho c) / A$, where, ρ, c, A are the air density , the velocity of the sound and the tube area respectively.

Therefore, as the sound wave propagates from the glottis to the lips, it will suffer reflections every time it encounters an interface ; that is, every time it enters a tube segment of different diameter (**Fig. 1.2**). So within each section, there will be forward and reverse travelling waves ; the forward travelling wave moves in the direction from glottis to the lips , and the reverse travelling wave moves in the direction from lips to the glottis. At the boundary between each section , some fraction of the forward travelling wave gets transmitted to the next section , and some fraction is reflected back. the same is true for the reverse travelling wave in each section. Multiple reflections will be set up in each segment and the tube will reverberate in a complicated manner depending on the number of segments and the diameter of each segment.

Let μ_m be the reflection coefficient at the boundary between *mth* and *(m−1)th* sections and u^+_m and u^-_m be the forward and reverse travelling waves respectively , & " τ " denote half the time required for a wave to travel from one end of the section to the other. It can be shown that

$$u^+_{m-1}(t+\tau) = \mu_m u^-_{m-1}(t-\tau) + (1+\mu_m)\, u^+_m(t-\tau) \qquad (1.7)$$

and

$$u^-_m(t+\tau) = (1-\mu_m)\, u^-_{m-1}(t-\tau) - \mu_m u^+_m(t-\tau) \qquad (1.8)$$

The above two relationships are described by a **lattice filter** , with the lattice parameters being equal to μ_i ' s , the reflection coefficients of

the acoustic tube model of the vocal tract (Fig. 1.3). For this lattice filter structure, the delay time 2τ of each section is taken to be a unit delay. As the shape of the vocal tract changes during utterances of different sounds, the cross sectional area of each section of the model, or equivalently the reflection coefficients μ_m, are modified.

The speech waveform coming out of the tube is recorded and the tube parameters such as the reflection coefficients and the width of each section are extracted after deconvolution.

1.5.2 MODELING OF SEISMOGRAM [2], [9], [17], [20], [21]:

The mathematical model for the real world seismic deconvolution problem is three - dimensional. This three - dimensional subsurface model is quite difficult to analyze ; it is still in research stage. So a one - dimensional subsurface model is considered. First , the fundamentals of seismic data processing are described. Then the modeling of the earth is discussed.

1.5.2.1 *Seismic data processing* :

Deposits of petroleum and natural gas are located in reservoirs deep below the surface of the earth. In order to find these resources geophysical exploration methods are used. Exploration seismology may be broadly divided into the *reflection seismology* and the *refraction seismology*. Most of the exploration of petroleum and natural gas is done by reflection seismic methods.

Reflection Seismology :

A source at the surface is set off. The source may be a

dynamite explosion or dropping of a weight , or some other means to transmit seismic energy into the earth. The seismic waves travel from the source to the subsurface layers within the earth where they are reflected. The reflected seismic waves are then recorded by instruments on or just below the surface. This recorded signal is known as a ***seismic trace or seismogram***. The earth's sedimentary layers are approximately horizontal , but they do have features such as anticlines that can serve as traps for petroleum and natural gas.

Each seismic trace is a time series made up of reflected waves together with various interfering waves and noise (Fig 1.4). The desired events are the primary reflections and the undesirable interfering waves are due to the multiple reflections. In order to suppress this interferance, redundancy in data aquisition is incorporatd. So in reflection seismology , seismic data are collected in a special way. A single *impulse source* is used along with many receivers, which are spaced equally along a line, for recording the seismogram. The source is activated and the traces are recorded. Then the entire configuration of the source and the receivers is moved by a distance equal to one half the distance between two receivers , and another set of seismograms are recorded for second source excitation. This procedure is repeated a number of times. By moving the configuration in such increments, each depth point is covered several times , and thus a multiple coverage of a depth point is obtained. Recording of seismic data by a multiple coverage scheme introduces considerable amount of overlap or redundancy. All the traces thus recorded , can be sorted out (or gathered) into a group such that all traces within that group have a *common depth point*

This is known as CDP gathering (Fig 1.5).

Next, some static and dynamic corrections are made to the seismic traces, in each group. A common depth - point gather or group is taken. Because of lateral variations in the thickness of the near - surface layers, each trace is corrected by a time shift refered to as *static correction*. The static correction has the effect of placing source and the receiver on a fictitious horizontal datum plane. The dynamic correction is to overcome the effect of offset between source and receiver. Increasing the offset increases the arrival time of a reflection from a given interface as a result of increased distance travelled by the wave. So the dynamic correction is the correction for the time of first arrival of the wave arriving at each receiver so as to convert each trace to the equivalent trace that would have been received if both the source and the receiver are at the same point (i.e. at a point directly above the common depth point of the group). In other words, under the appropriate dynamic corrections, the primary reflections of all the traces in the group will be in phase, thus making the corrected traces coherent. Then next step is to add all the traces in a group. By adding, the primary reflections, which are in phase they are amplified, whereas the random noise and the multiple reflections are suppressed. The resulting trace is equivalent to the one, which could have been obtained if the source and the receiver were at the same point, and is called " **normal incidence seismogram** ".

After the signal enhancement, the next step is to recover the information about the earth's subsurface. This can be done by ***deconvolution*** of the seismogram (i.e. the normal incidence seismogram).

1.5.2.2 ***Modeling of the earth*** :

There are two basic modeling approaches in seismic data processing.

1. Random model :

In the Random model , the depths and the reflectivities of the deep reflecting horizons are considered to be random in nature. This consideration is valid in seismology because, the widths of the different layers and their composition are different. As the reflection coefficient at the interface depends upon the the acoustic impedance ,which inturn depends upon the composition of two adjoining layers , the reflection coefficients can be considered to be random. The source signal is not a unit spike but a minimum or mixed delay signal called ***seismic wavelet***. The surface layers produce *reverberation* due to multiple reflections. Thus the seismogram is the convolution of the seismic wavelet , the random reflection coefficient series and the reverberation sequence.

2. Dynamic model :

The earth's geological layers are characterised as elements of a stratified system for the purpose of modeling the acoustic wave propagation in various layers. The dynamic model of the earth is based on the following facts : wave motion in each layer is characterised by two components travelling in opposite directions ; the upgoing wave in each layer is the superposition of the reflection of an downgoing wave in that layer and the transmission of an upgoing wave from the lower layer; and the downgoing wave in each layer is the result of the transmission of a downgoing wave from the upper layer and a reflection

of an upgoing wave in that layer (Fig 1.5). This concept is similar to that for a travelling wave in the acoustic tube , the model for vocal tract , in speech processing. Under the modeling constraints (namely normal incidence , one dimensional wave equation and homogeneous & isotropic horizontal layers) these physical facts can be employed to derive interrelationships which relate the upgoing and downgoing waveform from one layer to the other. These lead to **lattice - like structures** which give physical meaning to the acoustic wave propagation between different layers.

The seismic problem is somewhat different from the speech analysis. In seismology , it is not the transmitted wave that is experimentally accessible , rather it is the overall reflected wave. On the basis of this reflected wave , the layered structure (i.e the reflection coefficients) are to be extracted using deconvolution.

However the point to be emphasized is that both the dynamic modelling of the earth and the acoustic tube model of the vocal tract lead to the " lattice structures ", and hence the analysis is valid for both the applications. However it should be noted that the Random model is applicable only in seismic signal processing.

1.6 SCOPE OF THE WORK :

Each of the two deconvolution methods, viz., Homomorphic and Predictive , has particular advantages and limitations when used separately. In this thesis , an attempt has been made to combine the two methods to make use of the advantages of both. This approach is seen to result in significant improvement in the performance. The deconvolution

method is illustrated by means of synthetic examples for the two models viz., Random Model and Dynamic Model. The output of the system , for both minimum phase and maximum phase input is synthesised and then it is used to test the deconvolution algorithms. First the Predictive Deconvolution method and the Homomorphic Deconvolution method are used separately to recover the convolutional components. The results thus obtained are then compared with those obtained after combining both the methods. For the Dynamic model we then consider the recovery of the system parameters using System Identification Method and Dynamic Predictive Deconvolution method. The system identification method involves solution of Yule-Walker equations. However this method requires accurate a-priori knowledge of the order of the ARMA model of the system. The effect of additve white guassian noise on the deconvolution process is also studied. The Homomorphic signal processing is found to be more suitable for the recovery of the input signal to the system in the noisy as well as noiseless cases.

1.7 ORGANISATION OF THE THESIS :

The Thesis is divided into six chapters. In Chapter - 2, the computation of the synthetic trace , for the two models , Random and Dynamic , is discussed. The two models , as applied to speech and seismic signal processing is discussed. The modeling of a layered medium as a lattice filter is next discussed.

In Chapter-3, the two deconvolution methods viz., predictive and homomorphic are discussed. The deconvolution of the synthetic traces using the two methods is illustrated. The problems associated with each

method are studied. The predictive deconvolution involves the solution of a set of normal equations. Levinson's algorithm is used to solve the normal equations. A strategy to determine the exponential weighting of the synthetic trace, so as to make the system impulse response a minimum phase sequence is proposed.

In Chapter-4, it is shown that there is a significant improvement in the results when the two methods – predictive and homomorphic – are combined. The all-pass filter for a mixed phase signal which is a major requirement in the predictive deconvolution method is determined using Homomorphic filtering. Both homomorphic filtering and the Dynamic Predictive Deconvolution method together are used to deconvolve the synthetic trace for the dynamic model.

The influence of additive white noise on homomorphic filtering is studied in Chapter-5. The thesis is concluded in Chapter - 6, which gives a summary of the results obtained .

The properties of the complex cepstrum are mentioned in Appendix.

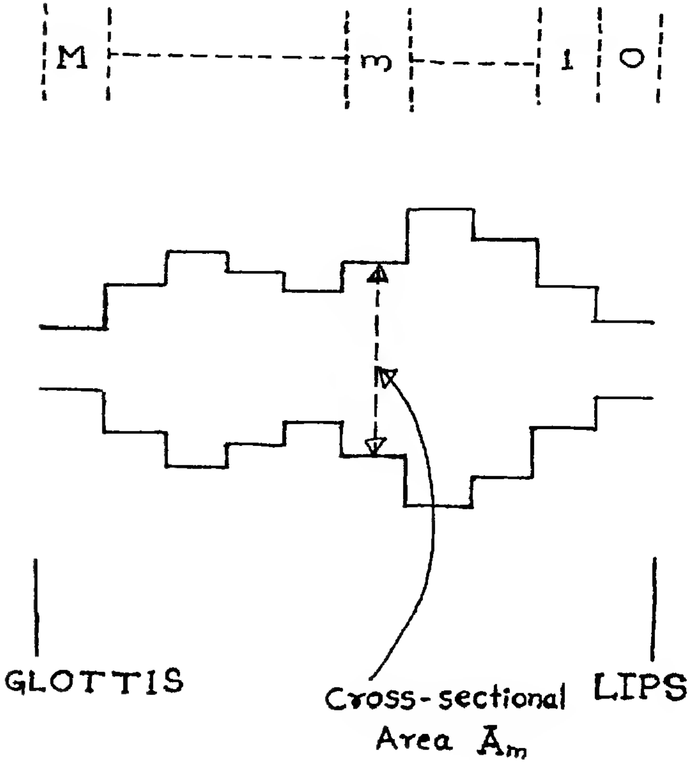


FIG 1·1

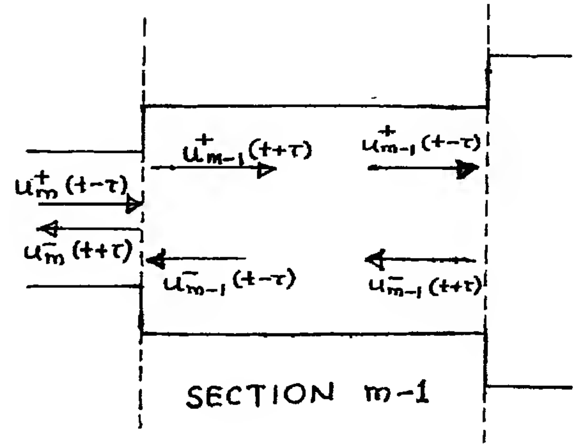


FIG 1·2

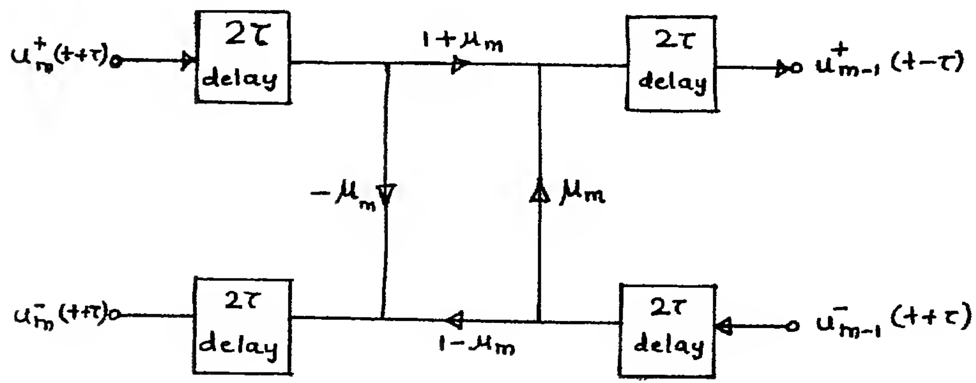
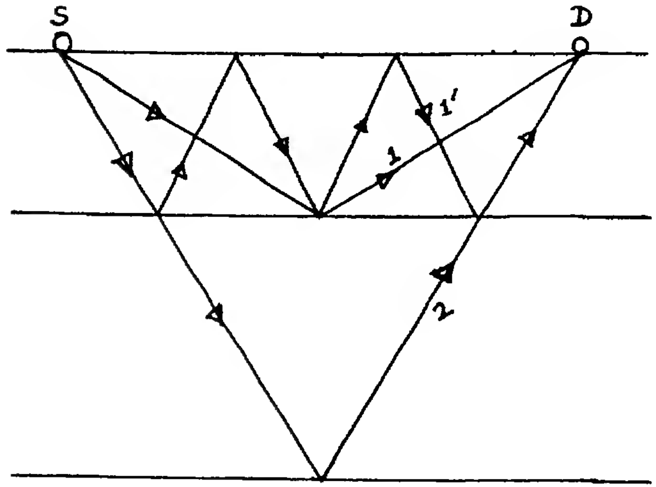


FIG 1·3



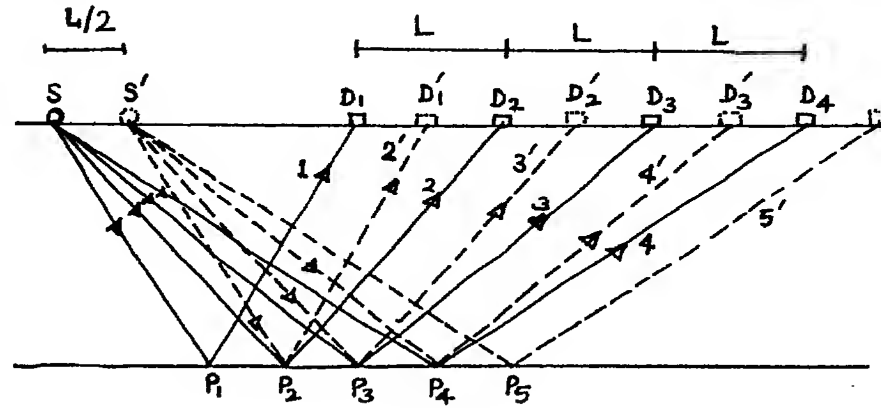
1, 2 — REFLECTED WAVE

1' — INTERFERING REVERBERATING-WAVE.

S — SOURCE

D — DETECTOR

FIG 1·4.



TRACES 2 & 2' HAVE A COMMON DEPTH POINT

TRACES 3' & 3' HAVE A COMMON DEPTH POINT

TRACES 4 & 4' HAVE A COMMON DEPTH POINT

FIG 1·5

CHAPTER - 2
SIMULATION

2.1 COMPUTATION OF SYNTHETIC TRACE :

A synthetic trace is the output of a known system for a known input. Computation of a synthetic trace requires the modeling of the system. After assuming some model for the system, the synthetic trace is computed by convolving a known input with the impulse response of the modeled system. This synthetic trace is then subjected to the deconvolution process. The deconvolution algorithms try to recover the input signal and the parameters of the modeled system from the synthetic trace. The relative performance of the deconvolution algorithms is studied by comparing the recoverd signals/parameters with those used for the computation of the synthetic trace. These simulation results are used to illustrate the improvement in the results of the deconvolution process when both predictive and homomorphic deconvolution methods are used together.

Two models for the system are considered and the synthetic traces are computed for these models. The models that are considerd are equivalent to those considerd for the vocal tract in speech analysis and the earth in seismic signal processing. But the parameters of the model that are considerd for the computation of the synthetic trace may not actually represent either the earth or the vocal tract.

The modeling of the system and the computation of the synthetic traces are discdussed in this chapter.

2.2 RANDOM MODEL :

The synthetic trace $t(n)$ is the convolution of three signals, and is given by,

$$t(n) = s(n) * r(n) * p(n) \qquad (2.1)$$

where, $s(n)$, $r(n)$, $p(n)$ are all real causal and stable.

The component $s(n)$ is either a minimum phase or a mixed phase signal. The sequence $r(n)$ is a random sequence [eq. 1.3]. The third component $p(n)$ is a periodic sequence given by,

$$p(n) = \sum_{k} \alpha^{k} [\delta(n-kN)] , \qquad (2.2)$$

where, $|\alpha| < 1$ and N is a positive integer and $k = 0, 1, 2, \ldots\ldots\ldots$

The above model is applicable in seismic signal processing, where, the seismogram is modeled as a convolution of the seismic wavelet (which is either minimum phase or mixed phase), the reflection coefficients of the layers of the earth and a reverberation wavelet (corresponding to $s(n)$, $r(n)$ and $p(n)$, in the above model). The reverberation wavelet is present when the source is buried under a strong reflector e.g. water in marine seismology. The reverberation effect will occur as a result of the trapping of most of the source energy in the water layer. The reflectin coefficients of the layers of the earth can be assumed to be random.

2.2.1 COMPUTATION OF THE SYNTHETIC TRACE FOR THE RANDOM MODEL

The computation of the synthetic trace is carried out in two stages. First the component signals $s(n)$, $r(n)$ and $p(n)$ are generated.

Then all the three components are convolved to get $t(n)$.

The signal $s(n)$ cam be either minimum phase or maximum phase. The minimum phase signal is represented by $s_{min}(n)$ and it is assumed to have the following z-transform [17] :

$$S_{min}(z) = \frac{1}{(1 - r.e^{j\theta} z^{-1})(1 - r.e^{-j\theta} z^{-1})} , \qquad (2.3)$$

where , $r = 0.9$, $\theta = \frac{\pi}{6}$.

The mixed phase signal is represented by $s_{mix}(n)$ and its z–transform is taken to be equal to,

$$S_{mix}(z) = \frac{S_{max}(z)}{(1 - r.e^{j\theta} z^{-1})(1 - r.e^{-j\theta} z^{-1})} , \qquad (2.4)$$

where , $r = 0.9$, $\theta = \frac{\pi}{6}$.

$$S_{max}(z) = (-0.5 - j0.5 - z^{-1})(-0.5 + j0.5 - z^{-1})(0.83 - z^{-1})(0.83 + z^{-1})(0.91 - z^{-1})$$

$$= 0.313 + 0.283 z^{-1} - 0.516 z^{-2} - 1.1 z^{-3} + 0.09 z^{-4} + z^{-5} \qquad (2.5)$$

The signals $s_{min}(n)$ and $s_{mix}(n)$, computed using the above z–transforms are shown in **Fig 2.1** and **Fig 2.2** respectively.

Figs. 2.2[a] and 2.2[b] show the minimum phase equivalent signal and the corresponding all-pass filter response , the details about which are given later in Chapter-3. The point , which is to be noted at this stage, is that the convolution of these two signals will give the mixed phase signal $s_{mix}(n)$.

TABLE - 1

n	r(n)	n	r(n)
0	0.4300	200	−0.1765
14	−0.0730	203	−0.1089
53	−0.2930	241	−0.1559
80	−0.1427	247	0.0150
105	−0.1465	275	−0.2028
110	0.2366	317	0.0300
115	0.2100	332	0.1277
162	0.1277	354	−0.0638
167	0.1500	377	−0.1540
187	-0.0500	381	0.1145
194	0.1070	431	−0.0526

The above random sequence is a modified version of the one given in [11]. The sequence r(n) is shown in **Fig. 2.3**. The autocorrelation function $\phi_{rr}(n)$ of r (n) for $n \geq 0$ is shown in the **Fig 2.5**. It can be seen that ,

$$|\phi_{rr}(n)| < 0.1\,\phi_{rr}(0) \quad \text{for } n \neq 0.$$

So r(n) can be considered to be approximately white.

The sequence p(n) is taken to be the following sequence :

$$p(n) = \sum_{k=0}^{13} (-0.387)^k \,.\, \delta(n - kN) \qquad (2.6)$$

The maximum value of "k" is taken to be 13 because the value of $(0.387)^k$ becomes comparatively neglegible for k>13. The **Figs. 2.4** and **2.9**

show the p(n) for the two values of N : N=22 and N=40.

The synthetic trace is computed by convolving the three components s(n), r(n), p(n). **Figs. 2.7, 2.8, 2.11** show the synthetic trace computed for three different combinations of the three component signals.

Fig. 2.7 : $t_1(n)$– Convolution of $s_{min}(n)$, r(n) and p(n) for N= 22.

Fig. 2.8 : $t_2(n)$– Convolution of $s_{mix}(n)$, r(n) and p(n) for N = 22.

Fig. 2.11 : $t_3(n)$– Convolution of $s_{min}(n)$, r(n) and p(n) for N = 40.

2 . 3 DYNAMIC MODEL :

A lossless layered medium which is described by the wave-equations and boundary conditions are considered for this Dynamic model. Specific applications of such layered media are : (1). horizontally stratified nonabsorptive earth with vertically travelling i.e. normal incident, plane compressional wave, (2). interconnection of lossless transmission lines, (3). the vocal tract modeling in speech analysis, etc.

A layered system consisting of K layers is shown in fig. 2.15. The thickness of each layer is considered to be equal. The input signal s(n) is applied at the interface#0. Let c_j be the normal incidence reflection coefficient associated with the j-th interface. Reflection coefficient c_j depends on the impedance of the materials at the 'j' th interface and $|c_j| < 1$. The output y(n), which is observed at the interface#0 contains information about the reflection coefficients. Since the characteristics of the layers are unknown, the main objective is to recover the reflection coefficients at each interface, which in turn are related to the impedances of the adjoining layers of the interface, by the relation,

$$c_j = \frac{z_j - z_{j+1}}{z_j + z_{j+1}} , \tag{2.7}$$

where z_j is the impedance of "j" th layer.

The model assumes normal incidence ; horizontal layers , where each layer is lossless, homogeneous and isotropic ; small strains ; and an one dimensional wave equation. Each layer is assumed to have the same one-way travel time for a wave propagation. This common one-way travel time in each layer is taken to be *one half a unit time*. The layers having different travel times are built up by inserting hypothetical layers , whose reflection coefficients are equal to zero.

The upgoing wave in each layer is the superposition of the reflection of a downgoing wave in that layer and the transmission of an upgoing wave from the lower layer. The downgoing wave in each layer is the result of a transmission of a downgoing wave from the upper layer and a reflection of an upgoing wave in that layer [Fig. 2.16]. Let $d_j(n)$ and $u_j(n)$ be the downgoing and upgoing waves respectively in the j-th layer. The waves at the interface "j" can be related to those in the $(j+1)^{th}$ layer [17] as follows :

$$(1-c_j)\,.\,d_{j+1}(n) = d_j(n-\tfrac{1}{2}) - c_j\,.\,u_j(n+\tfrac{1}{2}) \tag{2.8}$$

$$(1-c_j)\,.\,u_{j+1}(n) = u_j(n+\tfrac{1}{2}) - c_j\,.\,d_j(n-\tfrac{1}{2}) \tag{2.9}$$

Using these equations a lattice form can be established which implements the upgoing and the downgoing wave in any layer. This lattice form is shown in the Fig. 2.17. The above equations can be written in matrix form [17] as follows :

$$\begin{bmatrix} D_{j+1}(z) \\ U_{j+1}(z) \end{bmatrix} = \frac{z^{1/2}}{1-c_j} \begin{bmatrix} z^{-1} & -c_j \\ -c_j z^{-1} & 1 \end{bmatrix} \begin{bmatrix} D_j(z) \\ U_j(z) \end{bmatrix} \tag{2.10}$$

Expanding (2.10) , a relation between the upgoing and the downgoing waves in the layer#0 to those in (K+1)th layer can be written as :

$$\begin{bmatrix} D_{K+1}(z) \\ U_{K+1}(z) \end{bmatrix} = \frac{z^{K/2}}{T^K} \begin{bmatrix} P_K{}^R & Q_K{}^R \\ Q_K & P_K \end{bmatrix} \begin{bmatrix} D_0(z) \\ U_0(z) \end{bmatrix} \tag{2.11}$$

where,

$$\begin{bmatrix} P_k{}^R & Q_k{}^R \\ Q_k & P_k \end{bmatrix} = \prod_{s=1}^{K} \begin{bmatrix} z^{-1} & -c_s \\ -c_s\, z^{-1} & 1 \end{bmatrix} \tag{2.12}$$

and ,

$$T^K = \prod_{s=1}^{K} (1 - C_s) \tag{2.13}$$

To compute the output of this layered system for a unit impulse input , the boundary condition $u_{K+1}(n) = 0$ is assumed to hold. This corresponds to the case when there is no upgoing waves from the deepest layer i.e. the $(K+1)^{th}$ layer and implies that at the final interface , all the downgoing energy is absorbed.

The layered medium model is also applicable to speech analysis. The similarity between the modeling of the layered earth in seimic signal processing and the vocal tract in speech analysis is discussed in [4] which describes the vocal tract lip reflectance by an ARMA filter.

2.3.1 COMPUTATION OF THE SYNTHETIC TRACE FOR THE DYNAMIC MODEL

The reflection coefficients, c_n's for the dynamic model are taken to be such that $c_{n+22}=r(n)$, where $r(n)$, n=0,1,.....431 is the sequence given in Table–I with $c_0 \triangleq 0.9$. Thus two–way travel time in the layer#1 is taken

to be 22 units. The output is considered to be recorded just below the interface#0.

Using (2.12) the polynomials P_k and Q_k are next computed using the above values of c_j's . For the boundary conditions $U_{k+1}(z) = 0$ and for unit impulse input, i.e. $D_0(z)=1$, we get from the relation-(2.11) :

$$U_0(z) = - \frac{Q_K(z)}{P_K(z)} . D_0(z) \tag{2.14}$$

The sequence $u_1(n)$ is computed from the relation-(2.11).

Finally , the two synthetic traces $t_4(n)$ and $t_5(n)$ are computed by convolving $u_1(n)$ with the input $s_{min}(n)$ and $s_{mix}(n)$ given in section 2.2.1

The Figs. 2.12-2.14 show unit impulse response $u_1(n)$ and the synthetic traces $t_4(n)$ and $t_5(n)$.

A fast algorithm to compute the polynomials P_K and Q_K is suggested by Choate, [6] , which reduces the number of multiplications from K^2 to $K.(\ln K)^2$.

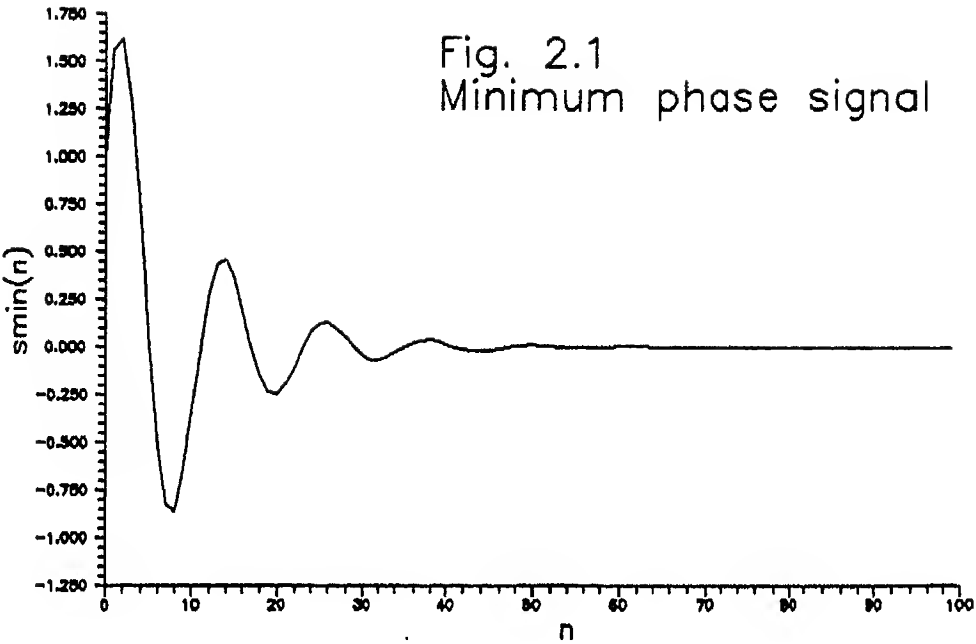


Fig. 2.1
Minimum phase signal

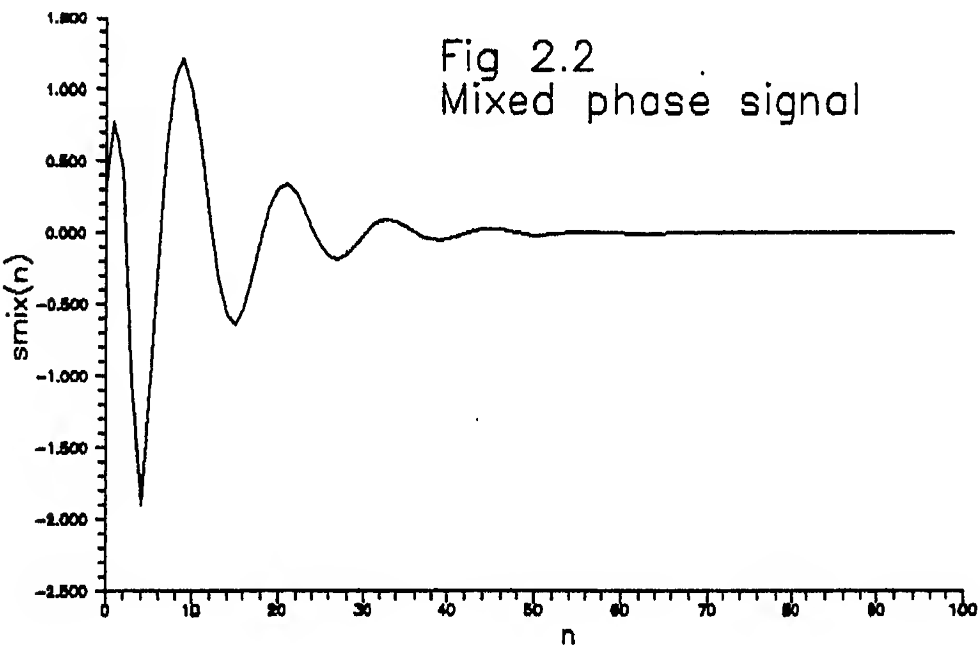


Fig 2.2
Mixed phase signal

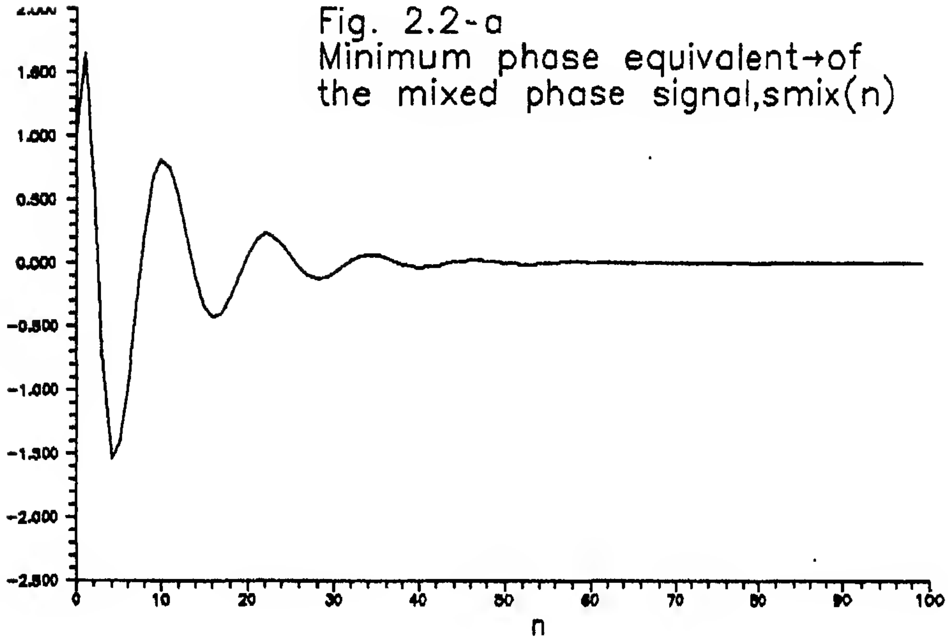


Fig. 2.2-a
Minimum phase equivalent→of the mixed phase signal,smix(n)

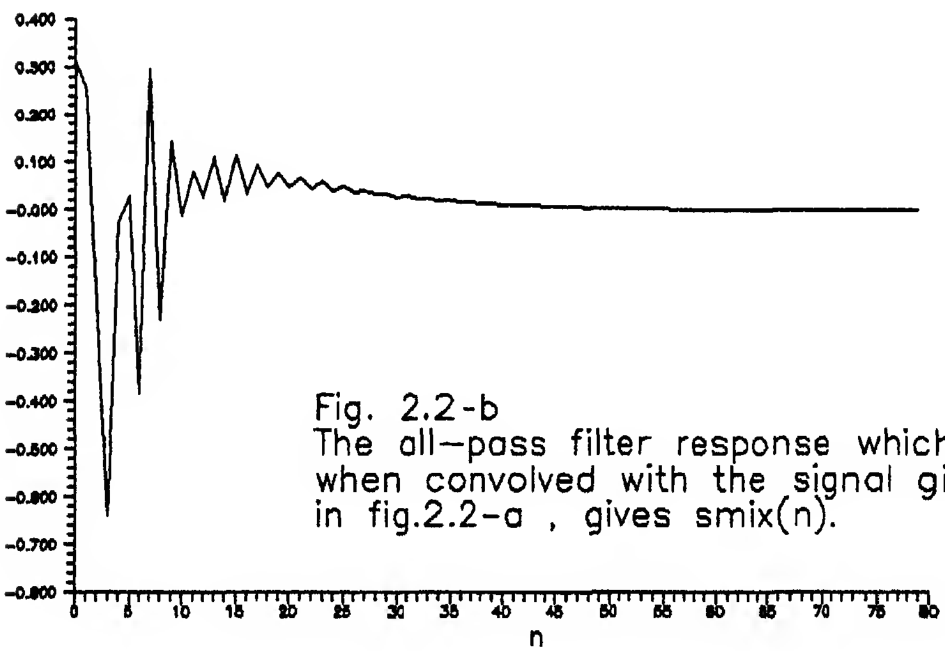


Fig. 2.2-b
The all-pass filter response which when convolved with the signal given in fig.2.2-a , gives smix(n).

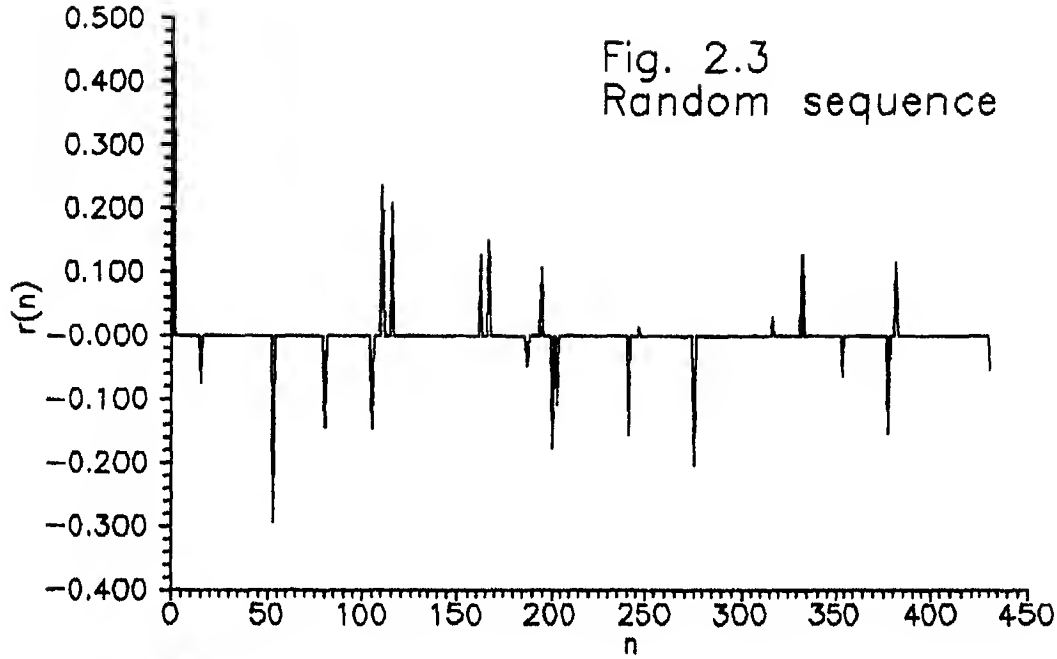


Fig. 2.3
Random sequence
r(n)
0.500
0.400
0.300
0.200
0.100
-0.000
-0.100
-0.200
-0.300
-0.400
0
50
100
150
200
250
300
350
400
450
n

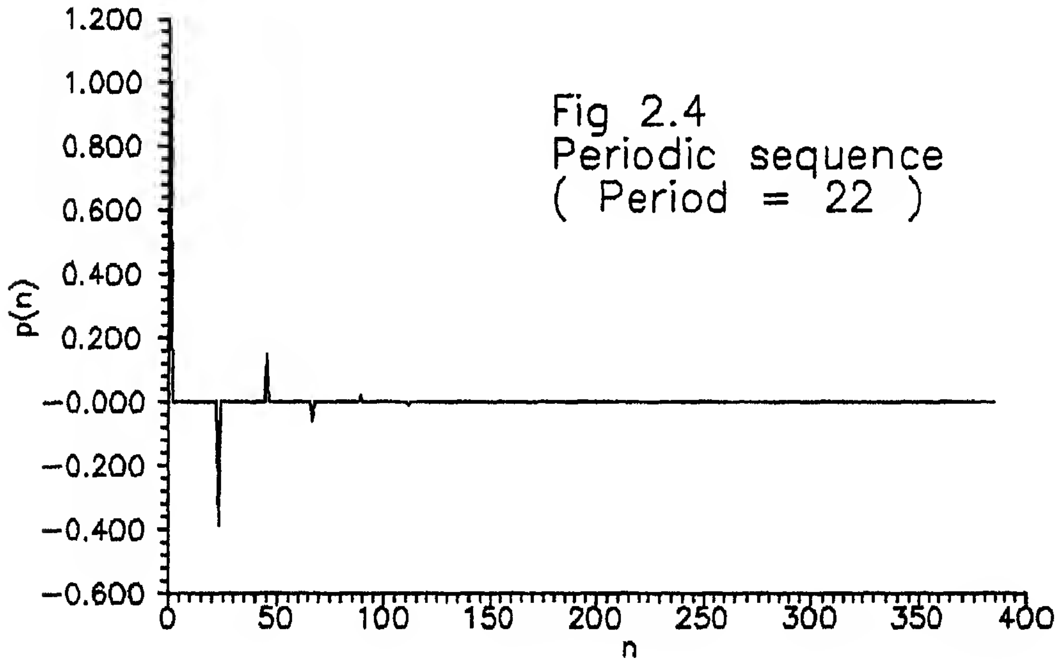


Fig 2.4
Periodic sequence
(Period = 22)
p(n)
1.200
1.000
0.800
0.600
0.400
0.200
-0.000
-0.200
-0.400
-0.600
0
50
100
150
200
250
300
350
400
n

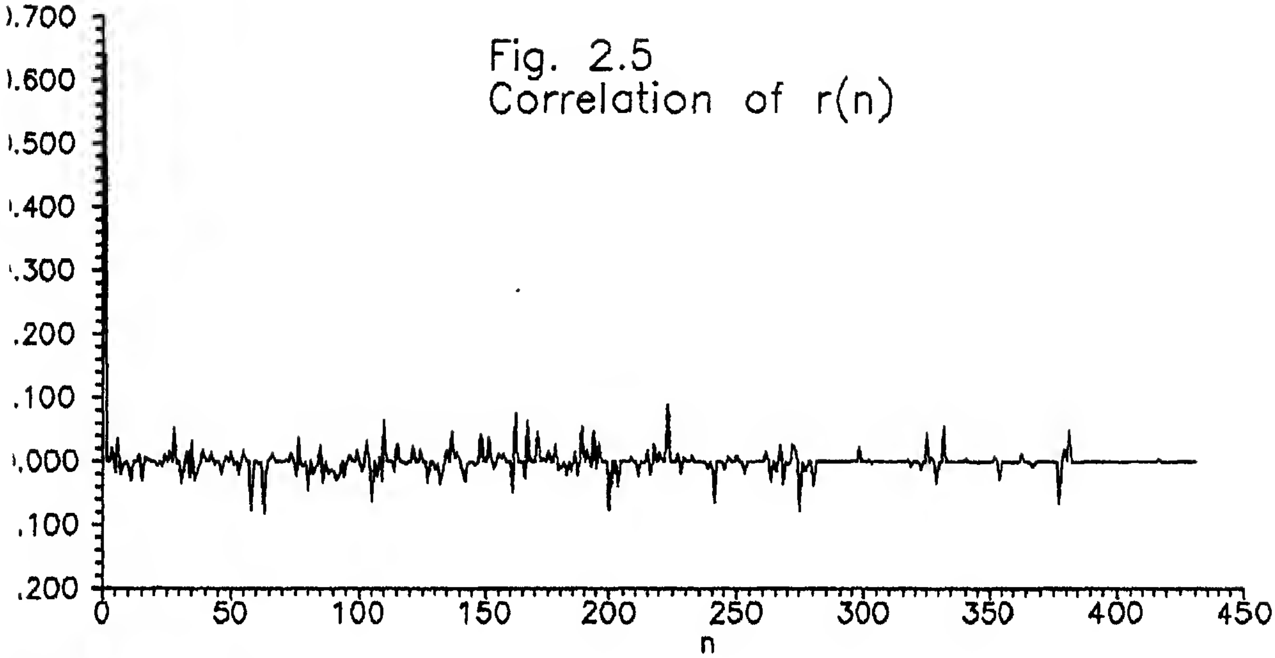


Fig. 2.5
Correlation of r(n)

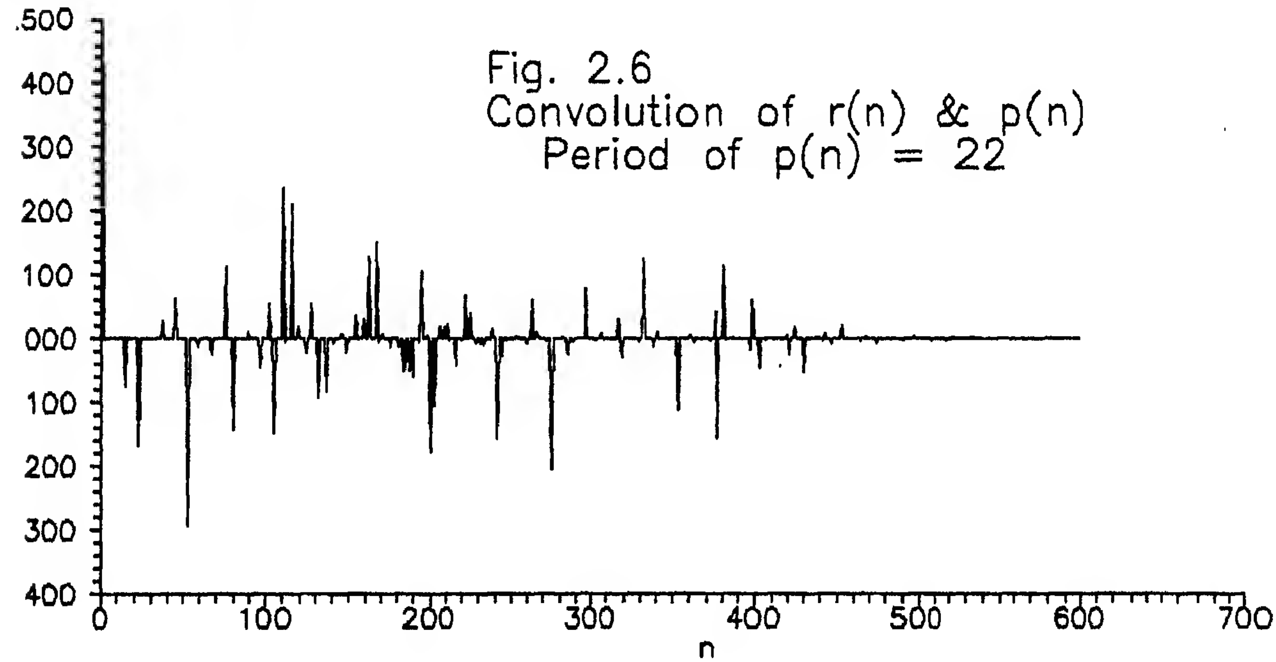
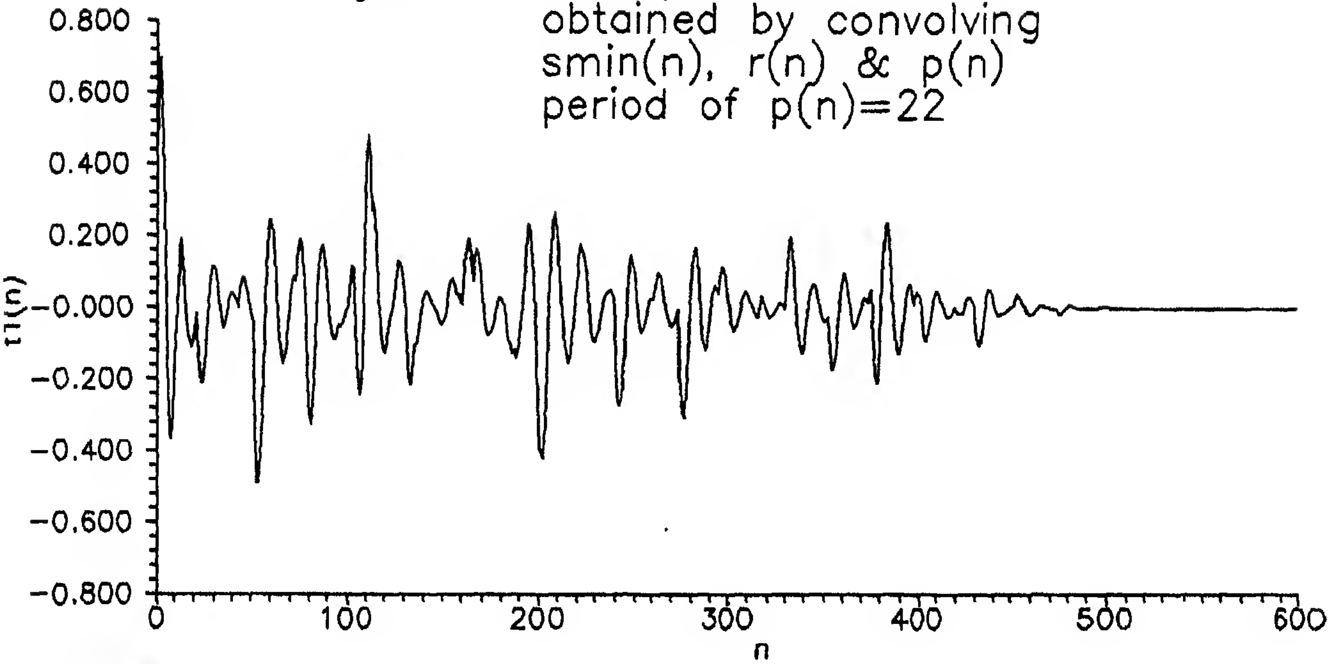


Fig. 2.6
Convolution of r(n) & p(n)
Period of p(n) = 22

Fig.2.7 The synthetic trace obtained by convolving smin(n), r(n) & p(n) period of p(n)=22



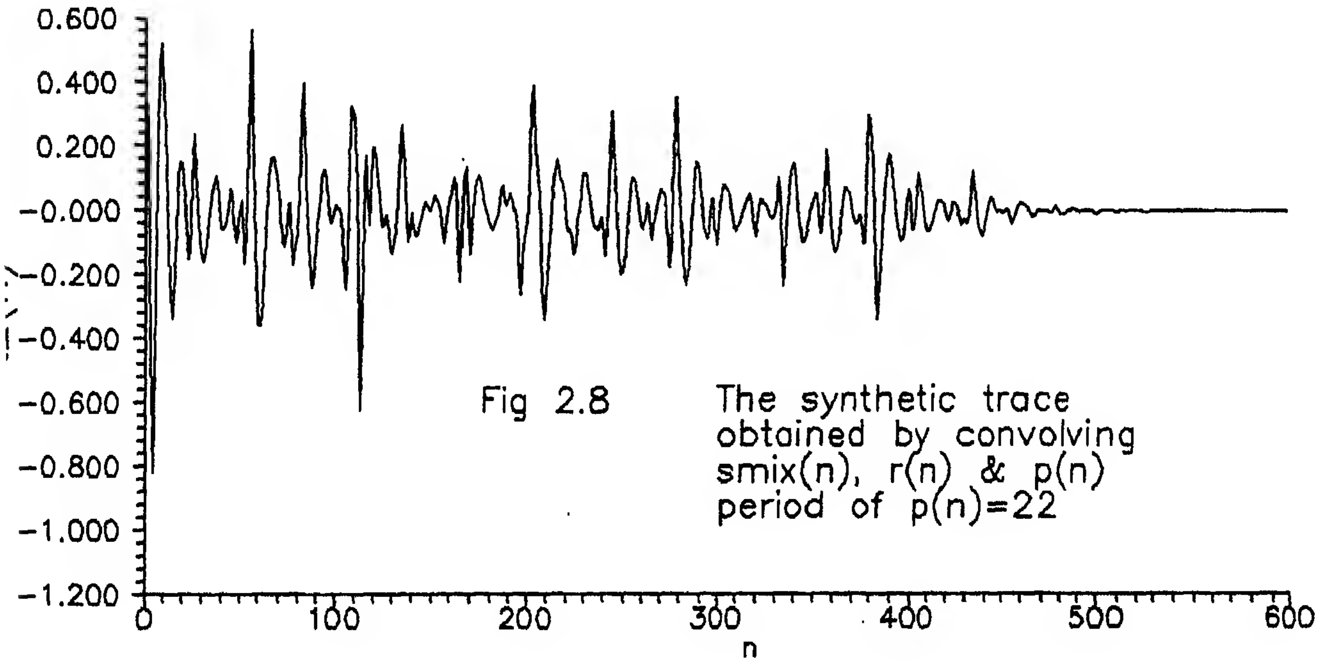


Fig 2.8 The synthetic trace obtained by convolving smix(n), r(n) & p(n) period of p(n)=22

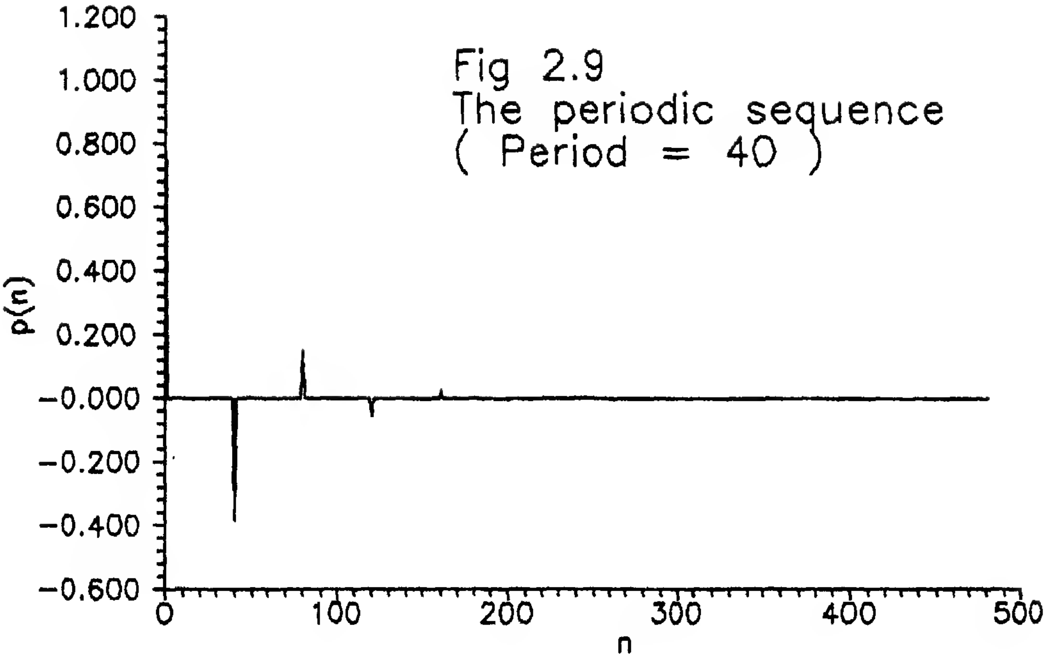


Fig 2.9
The periodic sequence
(Period = 40)
p(n)
1.200
1.000
0.800
0.600
0.400
0.200
−0.000
−0.200
−0.400
−0.600
0
100
200
300
400
500
n

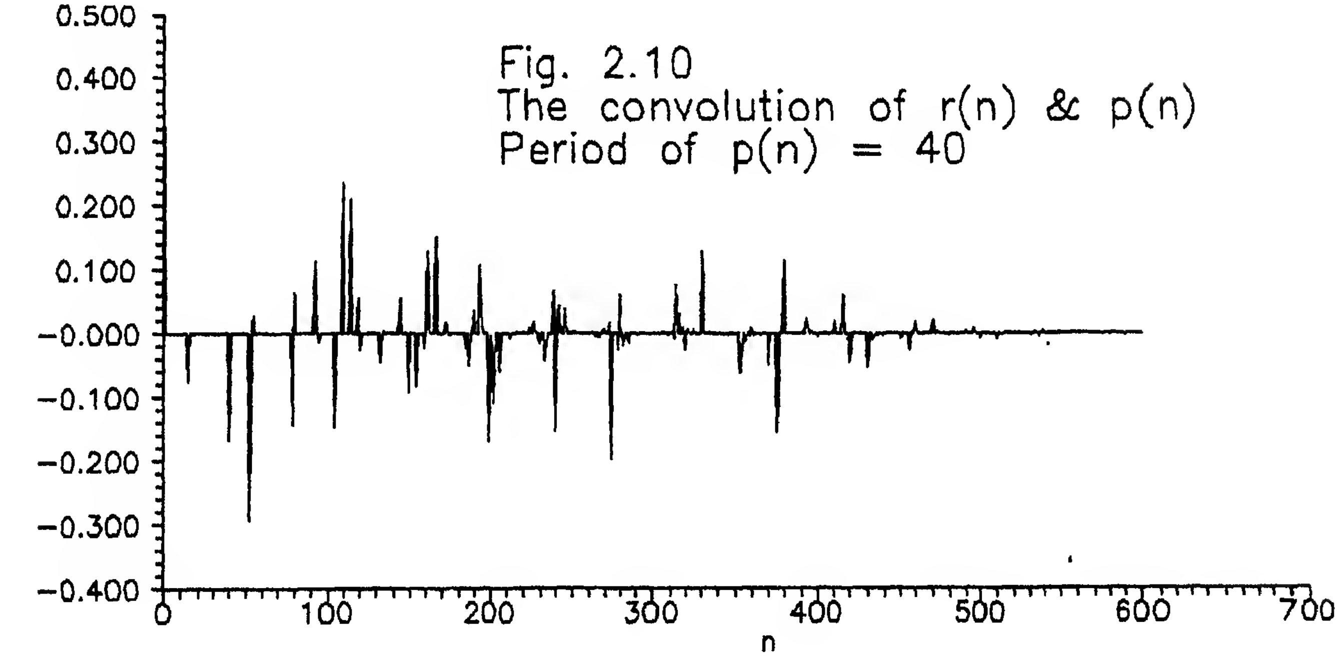


Fig. 2.10
The convolution of r(n) & p(n)
Period of p(n) = 40
0.500
0.400
0.300
0.200
0.100
−0.000
−0.100
−0.200
−0.300
−0.400
0
100
200
300
400
500
600
700
n

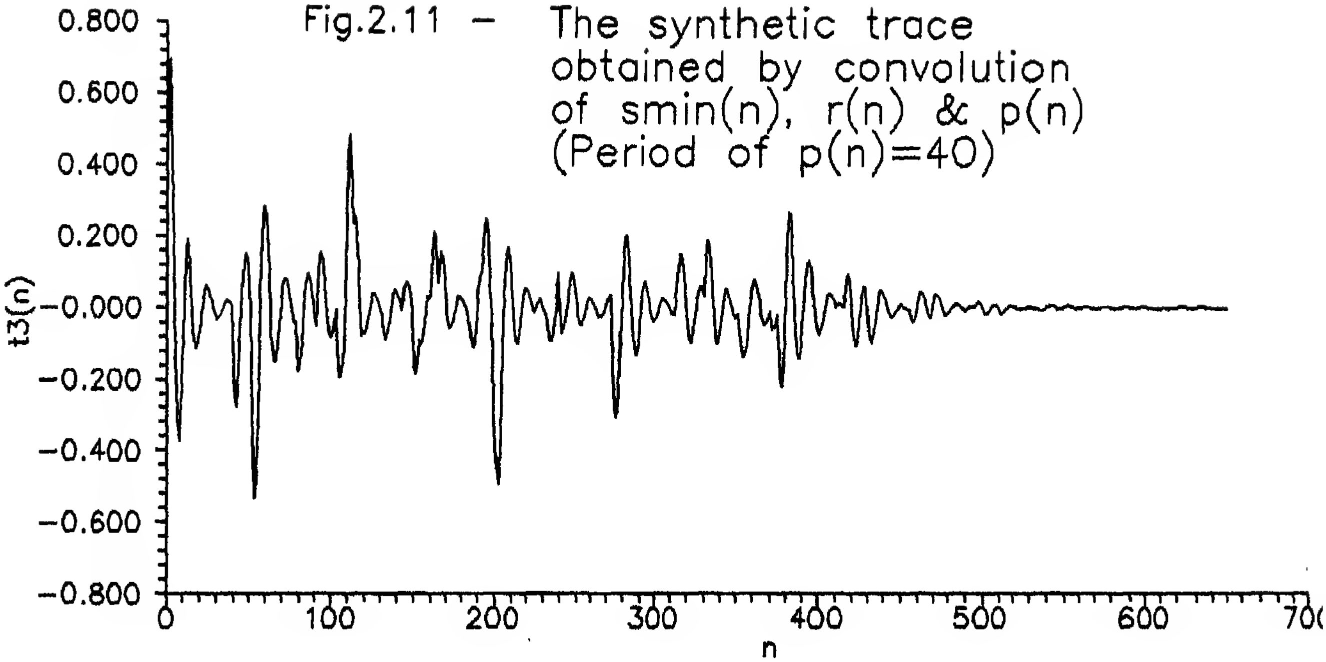


Fig.2.11 - The synthetic trace obtained by convolution of smin(n), r(n) & p(n) (Period of p(n)=40)

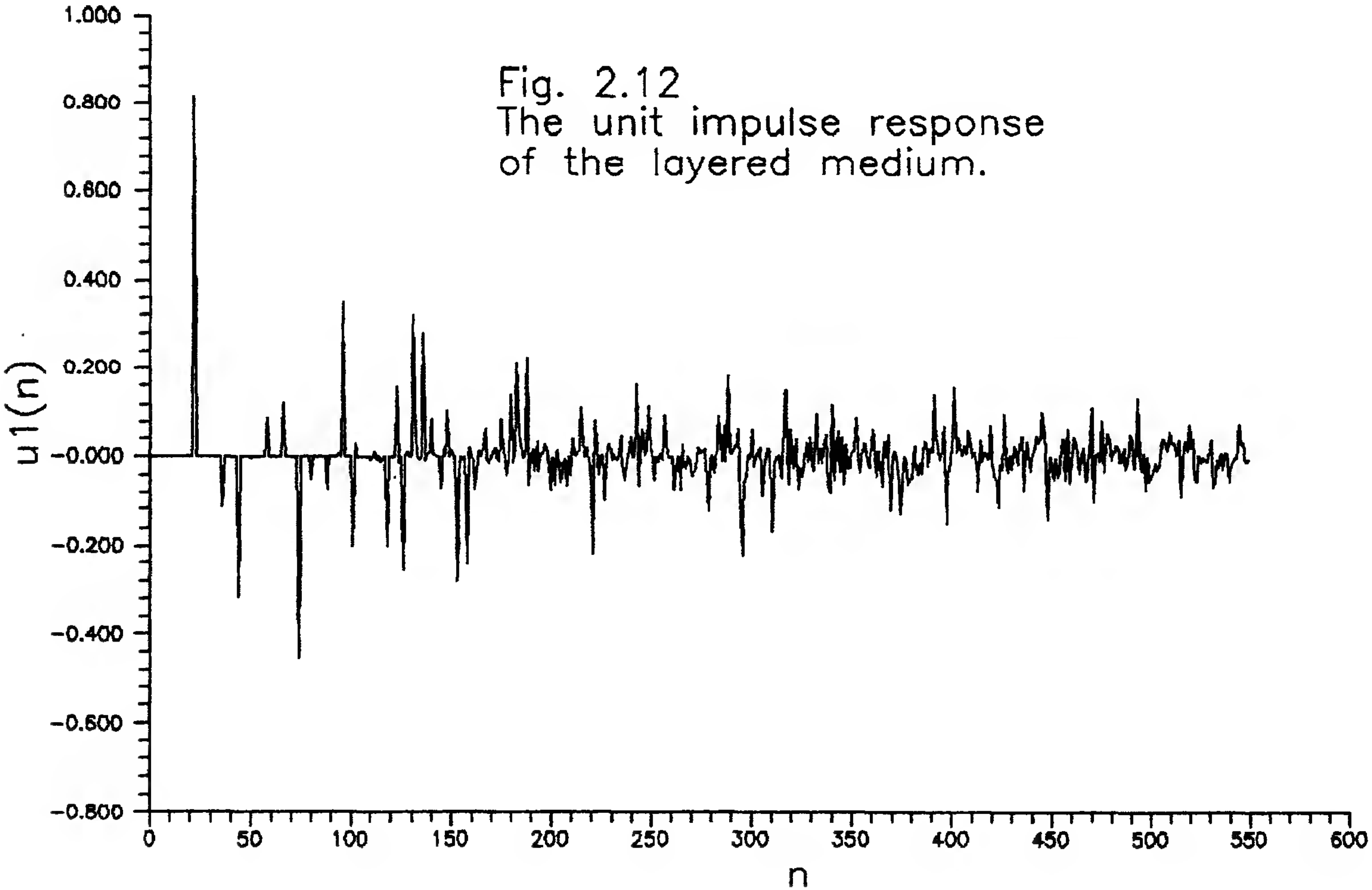


Fig. 2.12
The unit impulse response
of the layered medium.

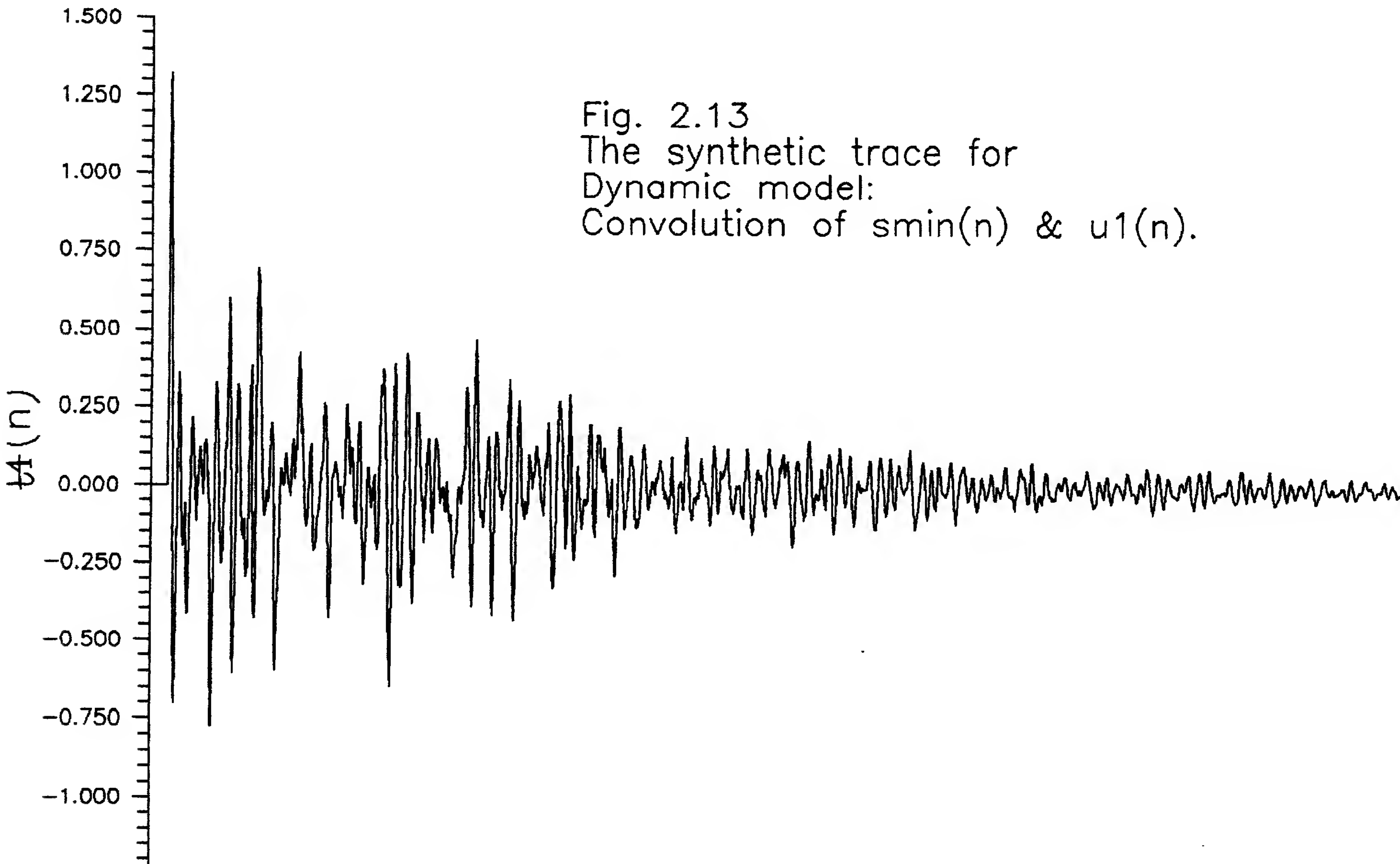


Fig. 2.13
The synthetic trace for
Dynamic model:
Convolution of smin(n) & u1(n).

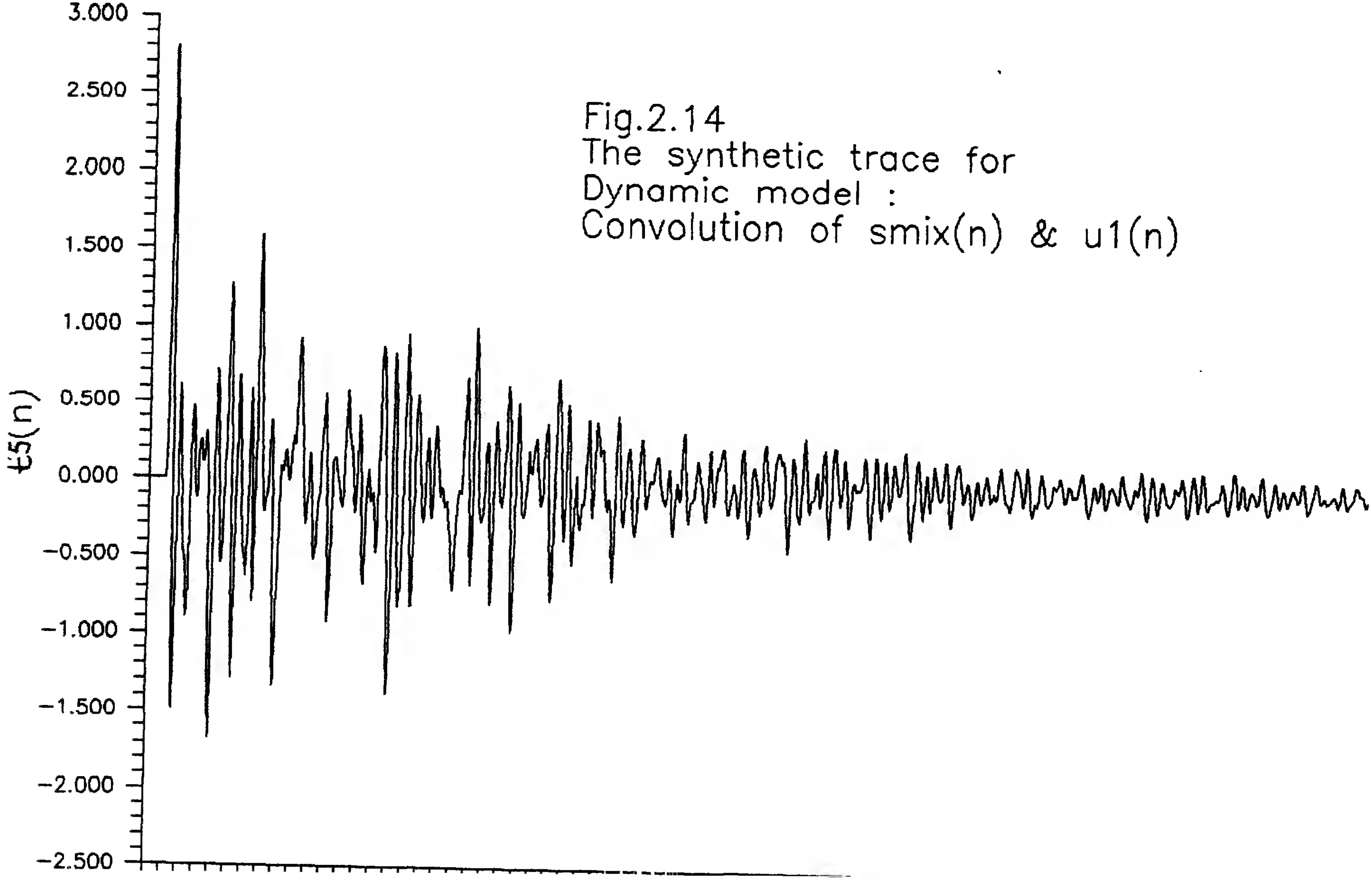
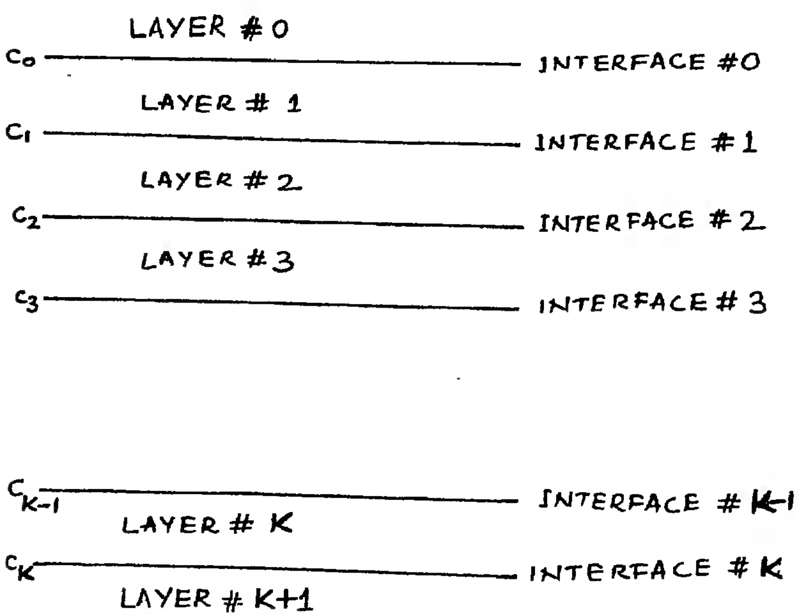


Fig.2.14
The synthetic trace for
Dynamic model :
Convolution of smix(n) & u1(n)



C_i — REFLECTION COEFFICIENT OF i-th INTERFACE.

FIG. 2.15 LAYERED MEDIUM

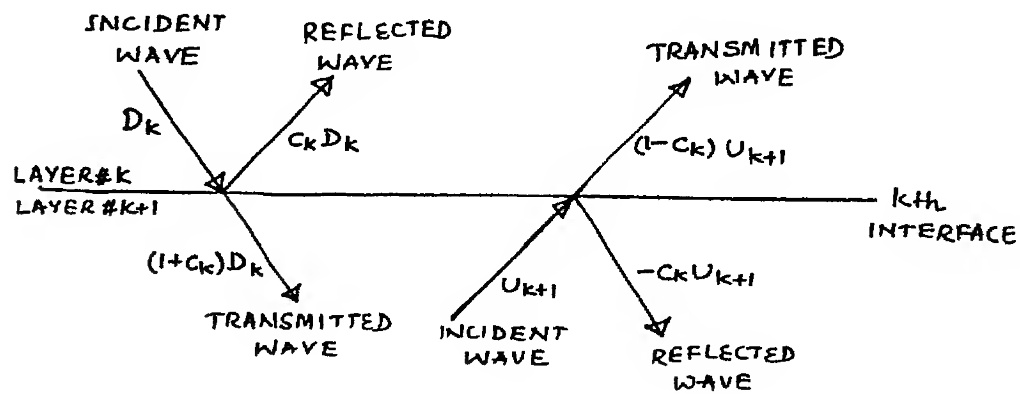


FIG 2.16

WAVES TRAVELLING THROUGH ADJACENT LAYERS

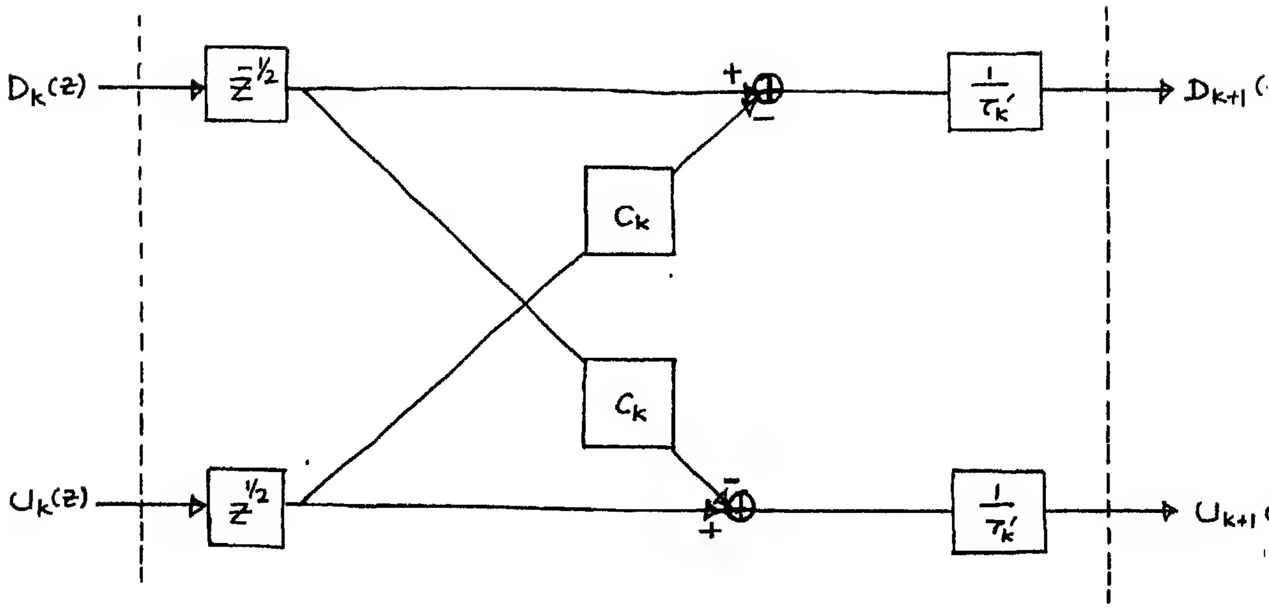


FIG 2·17 LATTICE FORM

CHAPTER 3

PREDICTIVE AND HOMOMORPHIC DECONVLUTION METHODS

The synthetic trace computed in the previous chapter is available for deconvolution. It is assumed that none of its convolutional components are known ; different deconvolution methods try to recover the convolutional components by making suitable assumptions. The recovered signals are then compared with the original signals and the efficiency of the different deconvolution methods are studied. The inference about the relative merits and demerits and the complexities associated with each deconvolution method is obtained. In this chapter the procedure is carried out for the Random model .

In this chapter, the predictive deconvolution and the Homomorphic deconvolution methods are used separately to recover the convolutional components. In chapter-4, we show that better results can be obtained when both the methods are combined together.

3.1 PREDICTIVE DECONVOLUTION :

Synthetic trace : $t(n) = s(n) * r(n) * p(n)$ (3.1)

Assumptions made:

1. $s(n)$ is a minimum phase signal.
2. $r(n)$ is a white noise sequence.

Consider the z-transform of the synthetic trace:

$$T(z) = S(z) \,.\, R(z) \,.\, P(z) \tag{3.2}$$

$$\text{or,} \quad T(z^{-1}) = S(z^{-1}) \,.\, R(z^{-1}) \,.\, P(z^{-1}) \tag{3.3}$$

Multiplying both the equations ,

$$T(z).T(z^{-1}) = S(z).S(z^{-1}) \,.\, R(z).R(z^{-1}) \,.\, P(z).P(z^{-1})$$

Taking the inverse z-transform of both the sides , we get ,

$$\phi_t(n) = \phi_s(n) * \phi_r(n) * \phi_p(n) \tag{3.4}$$

where , $\phi_s(n)$, $\phi_r(n)$, $\phi_p(n)$ respectively are the autocorrelation functions of $s(n)$, $r(n)$ and $p(n)$.

By assumption-2 , the sequence $r(n)$ is white , it follows that

$$\phi_r(n) = R.\delta(0)$$

where R is is a constant.

$$\phi_t(n) = R.\, \phi_s(n) * \phi_p(n) \tag{3.5}$$

Thus the autocorrelation of the synthetic trace is a scaled version of the autocorrelation of the convolution of the two convolutional components , $s(n)$ and $p(n)$. First let us consider the deconvolution of both $s(n)$ and $p(n)$ from the synthetic trace , so as to recover the sequence , $r(n)$. That is , it is required to find a filter $f(n)$, which when convolved with the synthetic trace , produces the sequence $r(n)$.

$$ry(n) * f(n) = r(n) * s(n) * p(n) * f(n) = r(n) \tag{3.6}$$

$$s(n) * p(n) * f(n) = \delta(n) \tag{3.7}$$

The signal, $s(n) * p(n)$ is not known. However, a scaled version of its autocorrelation , which is the autocorrelation of the synthetic trace itself [eqn. 3.5] , can be computed. The scaling does not affect the design of the filter f(n). A least- squares filter which minimises the error between the desired output and the actual output is designed. The expression for the error is given by,

$$E = \sum_{i} [\ z(n) - y(n)\ ]^2$$

where , $z(n) = \delta(n)$ is the desired output and $y(n) = s(n) * p(n) * f(n)$ is the actual output.

The minimisation of the error E , results in the Wiener Filter, $f(n)$, which can be obtained by solution of the normal equations :

$$\begin{bmatrix} \phi_t(0) & \phi_t(1) & \dots & \phi_t(L-1) \\ \phi_t(1) & \phi_t(0) & \dots & \phi_t(L-2) \\ \dots & \dots & \dots & \dots \\ \phi_t(L-1) & \phi_t(L-2) & \dots & \phi_t(0) \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \dots \\ f(L-1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix} \qquad (3.8)$$

where, $\phi_t(n)$ is the autocorrelation of the signal $s(n)*p(n)$, i.e. of the synthetic trace.

The normal equations are solved using Levinson's recursion algorithm which exploits the Toeplitz structure of the autocorrelation matrix. The total number of multiplications required to solve for a digital filter with L coefficients is proportional to L^2 for Levinson's recursion method , as compared to L^3 for Gauss-elimination method. Levinson's algorithm requires computer storage space , proportional to L rather than L^2 as in other methods. The filter $f(n)$ obtained from the solution of the above normal equations is next convolved with the synthetic trace , $t(n)$ to get back the random sequence $r(n)$.

Let $s(n)$ be a mixed phase signal. Any mixed phase signal can be represented as the convolution of its minimum phase equivalent signal and a corresponding all-pass filter: $s(n) = s_{meq}(n) * h_{ap}(n)$, where , $s_{meq}(n)$ is the minimum phase equivalent of $s(n)$, $h_{ap}(n)$ is the impulse response of the all-pass filter.

So the synthetic trace , $t(n)$ is written as :

$$t(n) = r(n) * h_{ap}(n) * s_{meq}(n) * p(n)$$

The filter $f(n)$ obtained by solving the eqns. 3.8 is such that,

$$f(n) * s_{meq}(n) * p(n) = \delta(n).$$

So, $f(n) * t(n) = r(n) * h_{ap}(n)$ is the recovered $r(n)$, which contains an all-pass component.

Now the sequence $s(n)$ is to be recovered. $s(n)$ can be either a minimum phase or a mixed phase signal. We now show that the $s(n)$ can be recovered only if it is minimum phase and the period of the sequence $p(n)$ is greater than the length of $s(n)$. Let N be the period of $p(n)$. Let the length of $s(n)$ be less than N. Then the first N samples of the signal $x(n)=s(n)*p(n)$ correspond to $s(n)$ itself. Now we design a $f'(n)$ which is a prediction operator with prediction distance equal to N. The output of the filter $f'(n)$ is given by,

$$y'(n) = \sum_i x(i)*f(n-i) = \bar{x}(n+N) \tag{3.9}$$

where $\bar{x}(n+N)$ is an estimate of $x(n+N)$.

The prediction operator is computed by minmising the error E given by,

$$E = \sum_i [\ z'(n) - y'(n)\ ]^2$$

where, $z'(n)=x(n+N)$ is the desired output and $y'(n)$ is the actual output.

The minimisation of E leads to the following matrix equation :

$$\begin{bmatrix} \phi_t(0) & \phi_t(1) & \cdots & \phi_t(K-1) \\ \phi_t(1) & \phi_t(0) & \cdots & \phi_t(K-2) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_t(K-1) & \phi_t(K-2) & \cdots & \phi_t(0) \end{bmatrix} \begin{bmatrix} f'(0) \\ f'(1) \\ \cdots \\ f'(K-1) \end{bmatrix} = \begin{bmatrix} \phi_t(N) \\ \phi(N+1) \\ \cdots \\ \phi(N+K-1) \end{bmatrix} \tag{3.10}$$

For a prediction operator $f'(n)$ of length K the corresponding prediction error operator $f''(n)$ with prediction distance N is :

$$f''(n) = \{ 1, 0, 0, \ldots\ldots\ldots, 0, -f'(0), -f'(1), \ldots\ldots\ldots -f'(K-1) \} \qquad (3.11)$$

Convolution of $x(n)$ with $f''(n)$ makes $x(n)$ vanish for the lags greater than N.

Thus if the duration of $s(n)$ is less than the period N of $p(n)$ then convolution of $x(n)$ with $f''(n)$ gives $s(n)$. We do not know $x(n)$ separately. The prediction error operator is hence convolved with the trace $t(n)$. As $r(n)$ is a white noise sequence, the autocorrelation of the resultant trace is the autocorrelation $\phi_s(n)$ of $s(n)$ itself. Next the AR filter coefficients $f(n)$ are computed from eqn. 3.8 with $\phi_t(n)$ replaced by $\phi_s(n)$. Then $s(n)$ is simply the impulse response of this AR filter. If $s(n)$ is a mixed phase signal then $s(n)$ recovered from the above method will be its minimum phase equivalent signal.

The predictive deconvolution fails to recover the signal $s(n)$ if the period , N is less than the duration of $s(n)$. If the wavelet is mixed phase , then only its minimum phase equivalent signal can be recovered. The Predictive deconvolution method requires $r(n)$ to be a perfect random sequence , which is not realisable.

3.1.1 ILLUSTRATION OF PREDICTIVE DECONVOLUTION :

(a). Consider the synthetic trace , $t_1(n)$.

1 . Find the autocorrelation of $t_1(n)$.

2. Solve the equatios , 3.8 , for $f(n)$ using Levinson's recursion. The filter length L is taken to be equal to 600.

3. Convolve $f(n)$ with $t_1(n)$ to recover $r(n)$. The recovered $r(n)$ is shown

in Fig. 3.1.2 along with its actual values [from Fig 2.3]. The deviation of the recovered r(n) from its actual values is due to the fact that r(n) is not a perfect white noise sequence. Hence in the above procedure, the minimum phase equivalent of r(n) also gets deconvolved along with s(n) and p(n).

4. Assume that the duration of s(n) is approximately 40 units [see Fig.2.1]. Use the method explained above to recover s(n). The recovered s(n) is shown in Fig. 3.1.1. The degradation of s(n) is due to the period of p(n) being less than 40 (equal to 22 for trace $t_1(n)$).

(b). Consider the synthetic trace, $t_2(n)$

The procedure given in (a) is repeated. The recovered s(n) and r(n) are shown in Figs. 3.1.3 and 3.1.4, respectively. Both the recovered signals deviate considerably from the actual signals, because s(n) is a mixed phase signal $s_{mix}(n)$ in the trace $t_2(n)$.

(c). Consider the synthetic trace, $t_3(n)$.

The autocorrelation of $t_3(n)$ is shown in Fig.3.1.5(a). The same procedure as in (a) is repeated for $t_3(n)$. The recovered s(n) is shown in Fig. 3.1.5 This recovered s(n) is a very good replica of the actual s(n), because in this case N=40.

Thus the problems which are not tackled by the Predictive Deconvolution are the determination of the all-pass filter when s(n) is mixed phase, and the restoration of the minimum phase equivalent of r(n), which gets deconvolved when r(n) is not a perfectly random sequence.

3.2 HOMOMORPHIC DECONVOLUTION :

Homomorphic decconvolution does not assume any of the convolved components to be minimum phase or perfectly random. However the quality of the results depend upon the degree of separatability of the component signals in the *"quefrency"* domain. Two convolutional components are well separated in the quefrency domain if the spectrum of one component is slowly varying or smooth and the other is rapidly varying. The efficacy of this method depends mainly on factors such as the duration of $s(n)$, the time difference between the first two nonzero samples of the sequence $r(n)$, the period of $p(n)$, the choice of the window width etc. The determination of the amount of the exponential weighting of the synthetic trace in order to make the sequence $r(n)$, minimum phase also plays an important role in the success of the Homomorphic deconvolution method. A trial & error method for the determination of the amount of the exponential weighting is proposed in this thesis. A brief review of the Homomorphic Signal Processing is given in the Appendix. The deconvolution method is explained below.

Consider the synthetic trace , $t(n)= s(n) * r(n) * p(n)$

The spectrum of $s(n)$ is smoothly varying and that of $r(n)$ and $p(n)$ are rapidly varying. So the complex cepstrum of $s(n)$ tends to be concentrated at the low quefrencies. If the signal $s(n)$ is a mixed phase signal , its complex cepstrum is nonzero for $n<0$. The sequence $p(n)$ is a minimum phase periodic sequence with period N. Hence its complex cepstrum is zero for $n< N$ and is a periodic sequence sequence with the same period N. The random sequence $r(n)$ in practice is a mixed phase

signal. So its complex cepstrum occupies the entire quefrency domain. The complex cepstrum of $r(n)$ can however, be made zero for $n<0$ by a suitable exponential weighting of the synthetic trace , such that $r(n)$ now becomes a minimum phase sequence. Let the synthetic trace be exponentially weighted by an amount, "a" , [$a<1$]. The z-transform of $a^n.t(n)$ is

$$T(az^{-1}) = S(az^{-1}) \, . \, R(az^{-1}) \, . \, P(az^{-1}) \qquad (3.12)$$

Thus if the synthetic trace is exponentially weighted , then each of its convolutional components also gets exponentially weighted by the same amount. Since $a<1$, all the zeros of the $R(az^{-1})$ can be made to lie within the unit circle by a proper choice of "a" , and thus $r(n)$ can be made minimum phase. Then the complex cepstrum of this minimum phase $r(n)$ is zero for $n< M+1$, where , M is the the number of samples between the first two nonzero samples of $r(n)$. The complex cepstrum $\hat{t}(n)$, of the exponentially weighted synthetic trace is next computed. Now , only the complex cepstrum of $s(n)$ is present near $n=0$. So $s(n)$ can be obtained low-pass ***"liftering"*** of the complex cepstrum , i.e. by multiplying $\hat{t}(n)$ by a window , $w(n)$, given by :

$$\begin{aligned} w(n) &= 1 \quad , \quad n< L \, ; \\ &= 0 \quad , \quad \text{elsewhere.} \end{aligned}$$

where , $L = \min(N, M+1)$. If $\hat{s}(n)$ is negligibly small for $n> L$, then $s(n)$ can be recovered successfully from $\hat{s}(n)$, no matter whether it is minimum phase or mixed phase. The signal $s(n)$ thus recovered is to be exponentially deweighted at the final stage of the reconstruction.

The inverse filter $f(n)$, for the deconvolution of $s(n)$ is obtained in a similar manner by using a window , $ww(n) = -w(n)$, where $w(n)$ is defined

above. The filter f(n) is exponentially deweighted and then convolved with the synthetic trace to deconvolve s(n). The signal s(n) can't be deconvolved using high-pass liftering because exponential deweighting will amplify the noise due to aliasing in the quefrency domain at the end of the deconvolved trace. This aliasing in the cepstral domain is always present because the complex cepstrum of a finite signal is of infinite duration. The aliasing in the quefrency domain can be reduced by taking the FFT and the IFFT length equal to about four times the length of the synthetic trace.

It is not possible to deconvolve p(n) from the trace, because the complex cepstra of r(n) and p(n) overlap. However, the sequence p(n) can be suppressed in the particular case in which the first few cepstral components of p(n) do not overlap with the cepstrum of p(n) and an a-priori knowledge of the period of p(n) is available. { For example, consider the complex cepstrum of the appropriately exponentially weighted synthetic trace $t_1(n)$. The first two nonzero cepstral components of r(n) are present at n=14 and n=53 whereas those of p(n) are at n=22 and n=44 }. Then the two cepstral values of $\hat{t}(n)$ at quefrency, n= N and n= 2N are made zero. This eliminates the first two samples of p(n) at n= N and n= 2N & reduces the remaining values of p(n) by 1/3 of their original value. Next, without any low-pass liftering , the synthetic trace is reconstructed after exponential deweighting. The exponential deweighting is done for only half the trace in order to reduce the amplification of noise due to aliasing. The contribution of p(n) is insignificant in this reconstructed trace . The inverse filter f(n) as computed above is then convolved with this reconstructed trace to recover the sequence , r(n).

The determination of the amount of exponential weighting and the width of the window are the two main problems associated with the homomorphic deconvolution. So far , no quantitative method for the determination of the amount of the exponential weighting is proposed [24]. In this thesis , a trial & error method for the determination of the required exponential weighting is proposed, which gives satisfactory results. This method is explained in the next section. The window-width is decided through observation.

3.2.1 DETERMINATION OF EXPONENTIAL WEIGHTING FACTOR:

Let "a" , where $a<1$, be the amount of the exponential weighting which is sufficient to make the sequence r(n) a minimum phase sequence. Then both the real and the complex cepstra of the sequence, $a^n . r(n)$ will be zero around zero and negative quefrencies and will be equal at higher positive quefrencies. This property is made use of in this method. Here, the high quefrency parts of the real cepstrum and the complex cepstrum of the synthetic trace , are compared for different amounts of exponential weighting , and the amount for which both the parts are same is chosen as the required exponential weighting. The values of "a" are chosen in a decreasing order starting from $a=1$. The cepstrum of the s(n) is concentrated only around zero quefrencies & the cepstrum at higher quefrencies is mainly due to r(n) and p(n). Hence this method works quite satisfactorily.

3.2.2 ILLUSTRATION OF HOMOMORPHIC DECONVOLUTION :

Consider the synthetic trace , $t_1(n)$.

1. Use the trial & error method to determine the exponential weighting required. The complex and real cepstra of $a^n.t(n)$ (a=0.992, 0.989) are shown in Fig 3.2.1(a) - 3.2.2(b). It can be seen that the high quefrency values of the complex cepstrum and the real cepstrum are the same for a=0.989 and are not same for a=0.992. So the required exponential weighting is taken to be 0.989. The complex cepstrum and the real cepstrum computed for this exponential weighting are used for deconvolution.

2. In **Fig.3.2.2(b)**, it can be seen that a strong peak appears at n=14, as the first cepstral value of r(n) . So the window w(n) is fixed through this observation , to be of length L=14.

3. The s(n) recovered from the real cepstrum is shown in **Fig. 3.2.3**. This recovered s(n) from the complex cepstrum of the synthetic trace will also be the same because the s(n) in the synthetic trace $t_1(n)$ is minimum phase.

4. The inverse filter f(n) of s(n) is computed by multiplying the cepstrum by a window ww(n)=−w(n).

5. Assuming that the period of p(n) is known a-priori to be equal to 22, the cepstral values for n=22 and n=44 are made zero. This will suppress remaining values of p(n) by about 1/3 . The synthetic trace is then computed back. It is then convolved with f(n) to get back r(n). The recovered r(n) is shown in **Fig 3.2.4**.

Consider the synthetic trace , $t_2(n)$.

The same procedure is repeated for this trace also. The complex and the real cepstra of $a^n.t_2(n)$ ($a=0.989$) are shown in **Figs. 3.2.5(a)** and **3.2.5(b)**. The minimum phase component of s(n) is recovered **(Fig.3.2.6)** from the complex cepstrum by using a window, w(n) given by,

$$w(n) = 1 \quad , \quad n < L ;$$
$$= 0 \quad , \quad \text{elsewhere.}$$

The maximum phase component of s(n) obtained from the negative quefrencies is shown in **Fig. 3.2.7**. Convolution of these two components gives the mixed phase s(n) (**Fig. 3.2.8**). The s(n) recovered from the real cepstrum is shown in **Fig. 3.2.9**. This is the minimum phase equivalent of the $s_{mix}(n)$.

Although the homomorphic deconvolution does not make use of any of the basic assumptions regarding the phase of s(n) and the randomness of r(n) , nevertheless it is a deterministic processing method. Hence , there is more uncertainity about the quality of the results obtained through this deterministic procedure. Choice of the width of the window and the amount of exponential weighting should be quite accurate. Slight deviation from the exact value of these two parameters yields poor results .

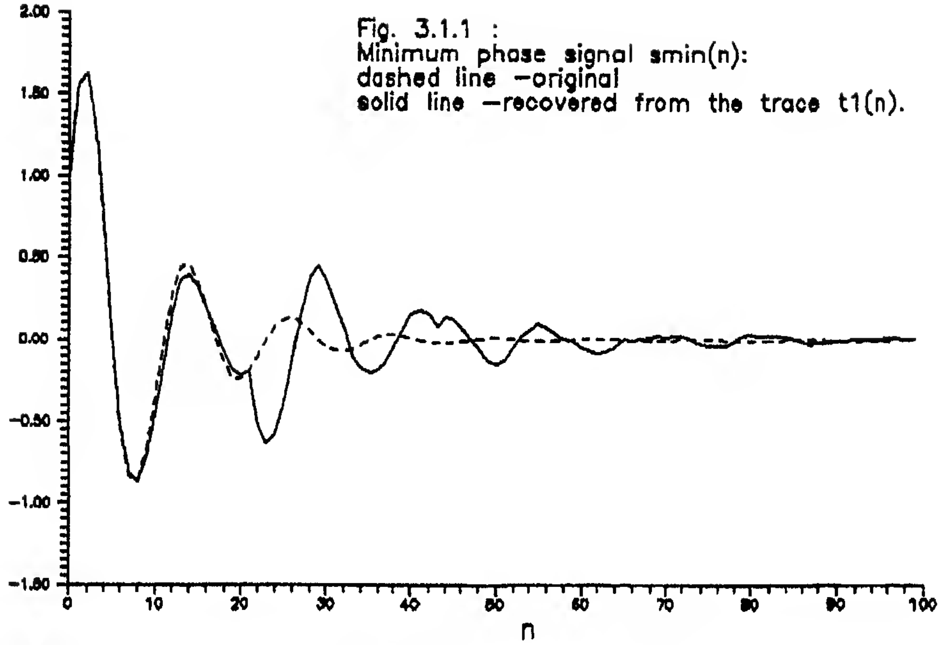


Fig. 3.1.1 :
Minimum phase signal smin(n):
dashed line —original
solid line —recovered from the trace t1(n).

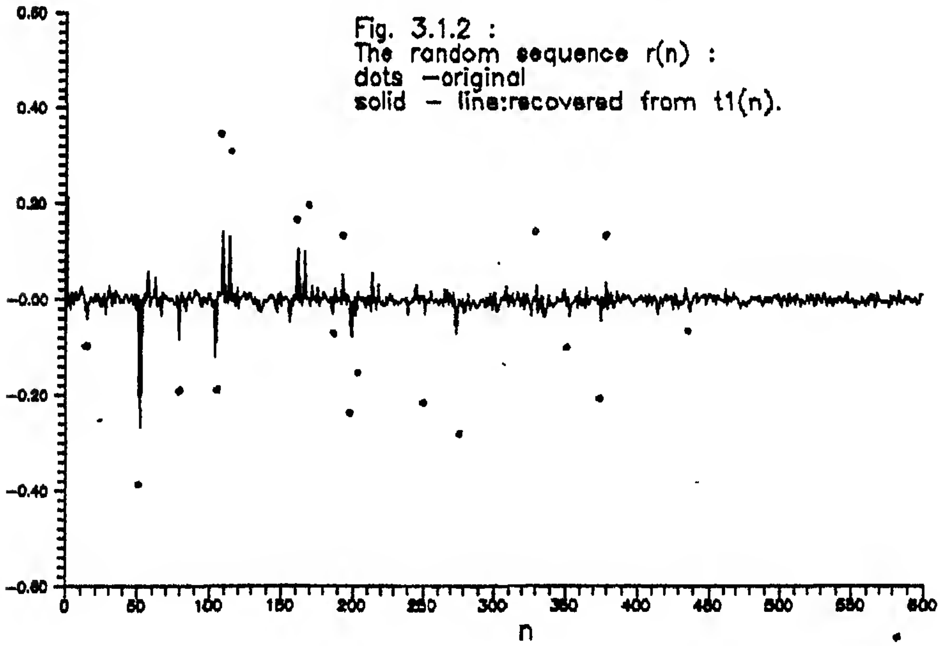


Fig. 3.1.2 :
The random sequence r(n) :
dots —original
solid — line:recovered from t1(n).

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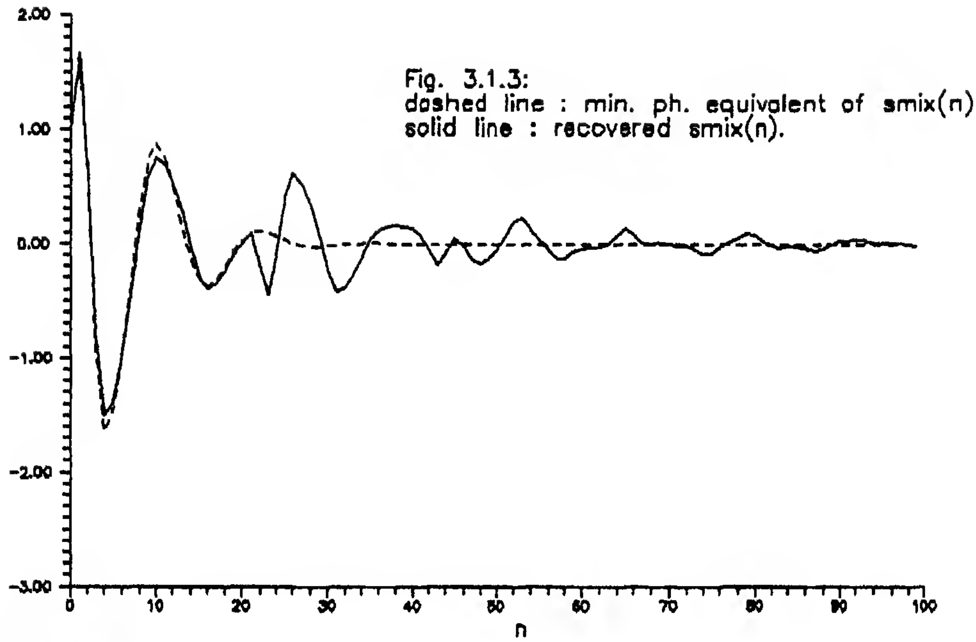


Fig. 3.1.3:
dashed line : min. ph. equivalent of smix(n)
solid line : recovered smix(n).
n

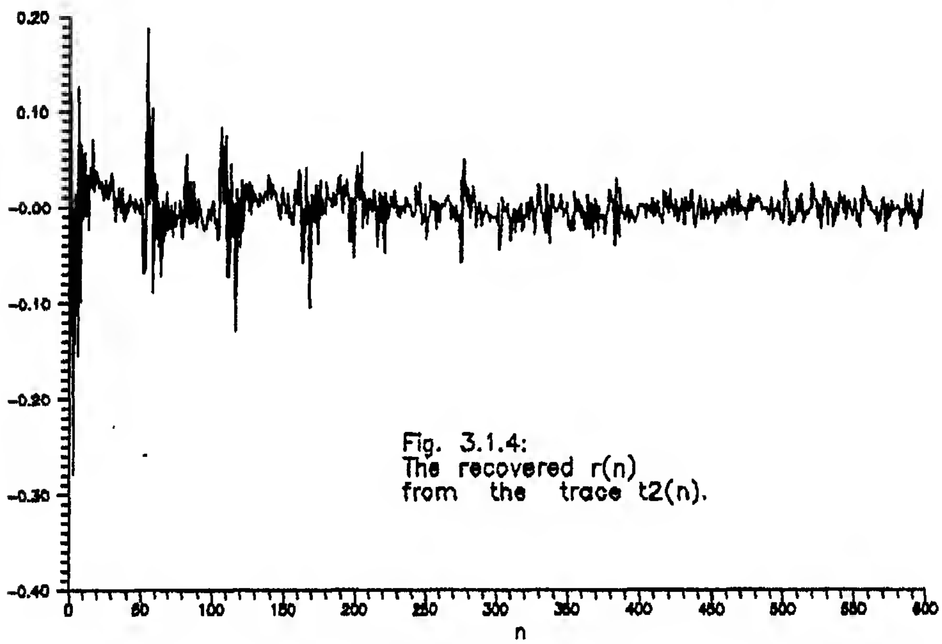


Fig. 3.1.4:
The recovered r(n)
from the trace t2(n).
n

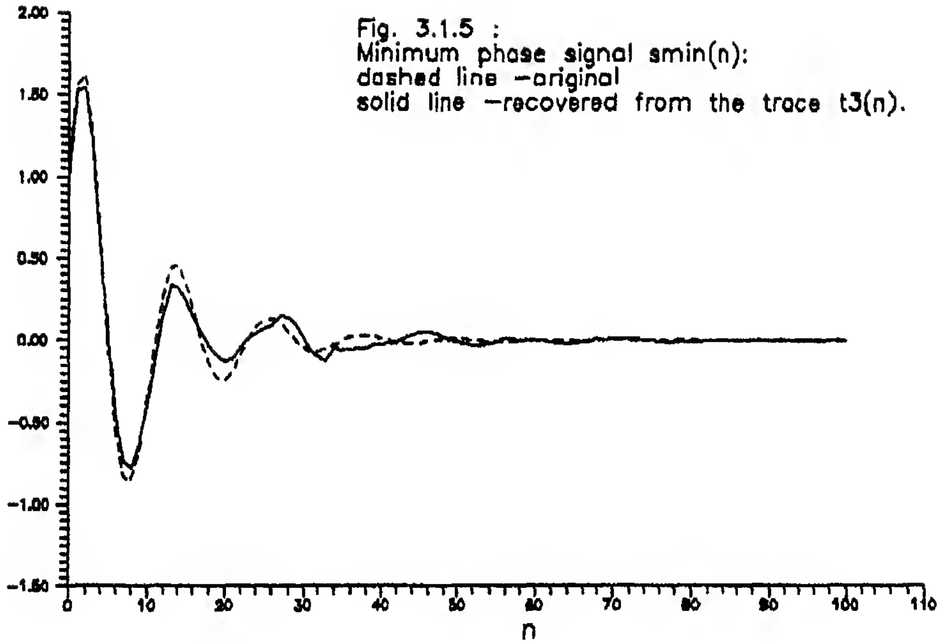


Fig. 3.1.5 :
Minimum phase signal smin(n):
dashed line -original
solid line -recovered from the trace t3(n).
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
0
10
20
30
40
50
60
70
80
90
100
110
n

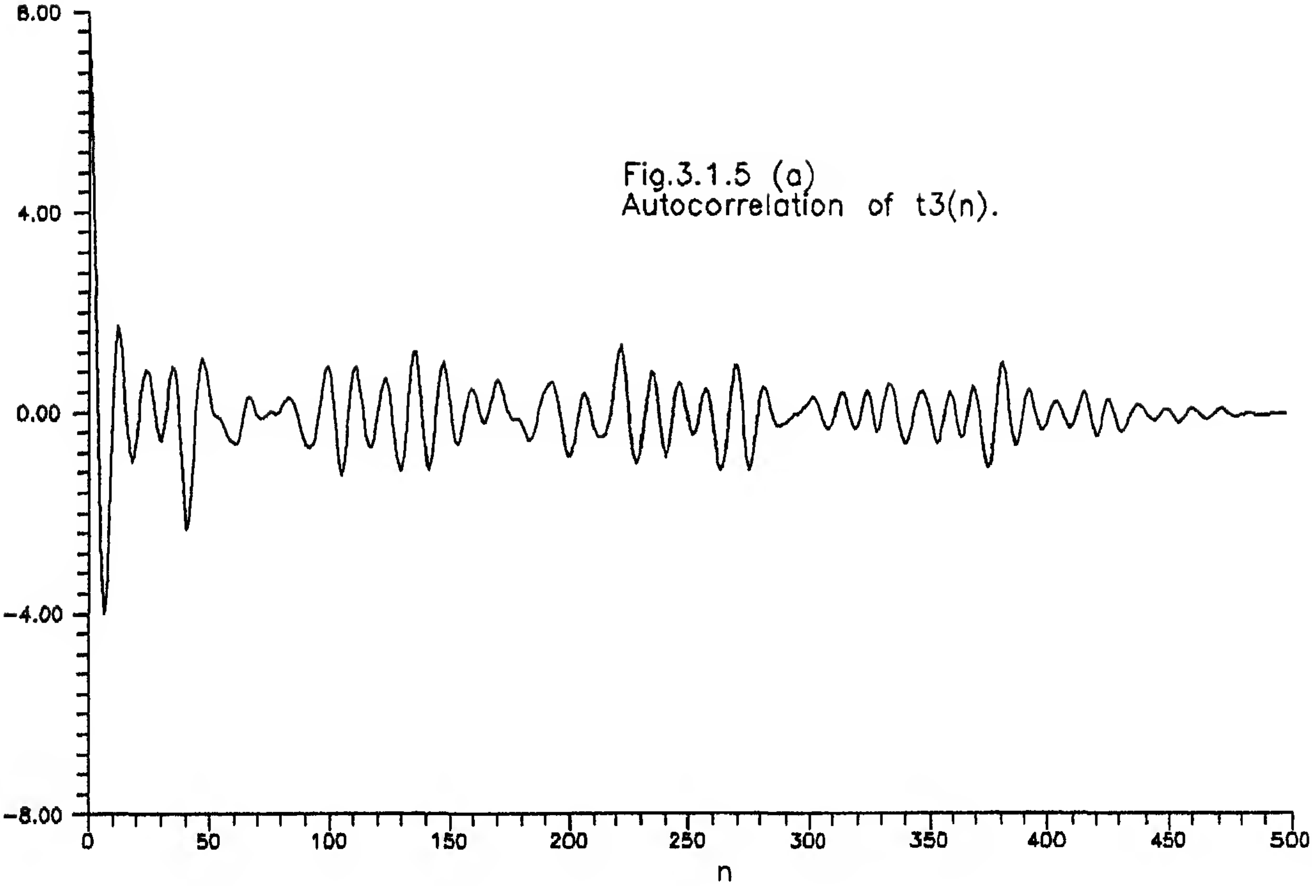


Fig.3.1.5 (a)
Autocorrelation of t3(n).

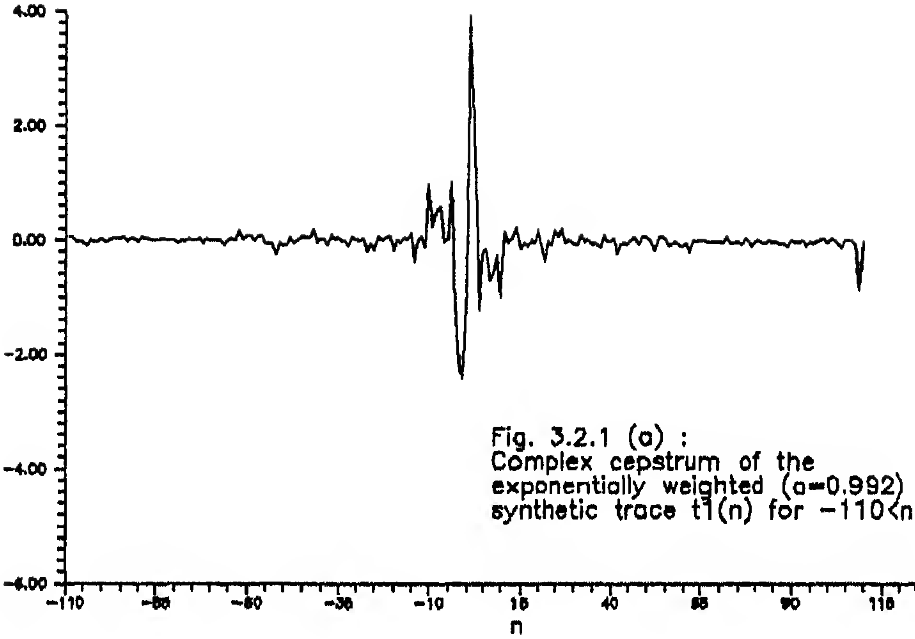


Fig. 3.2.1 (a) :
Complex cepstrum of the
exponentially weighted (a=0.992)
synthetic trace t1(n) for $-110<n<110$

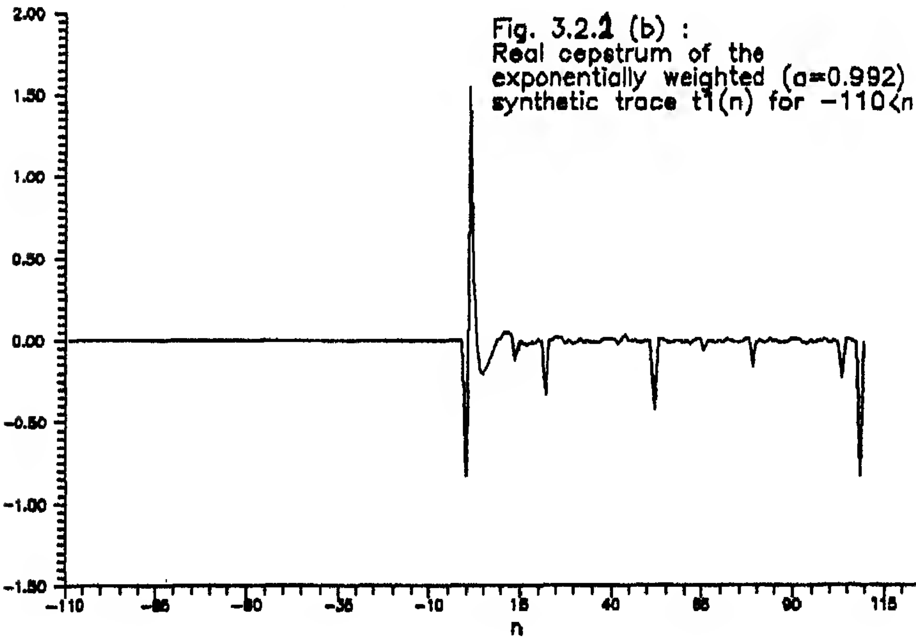


Fig. 3.2.1 (b) :
Real cepstrum of the
exponentially weighted (a=0.992)
synthetic trace t1(n) for $-110<n<110$

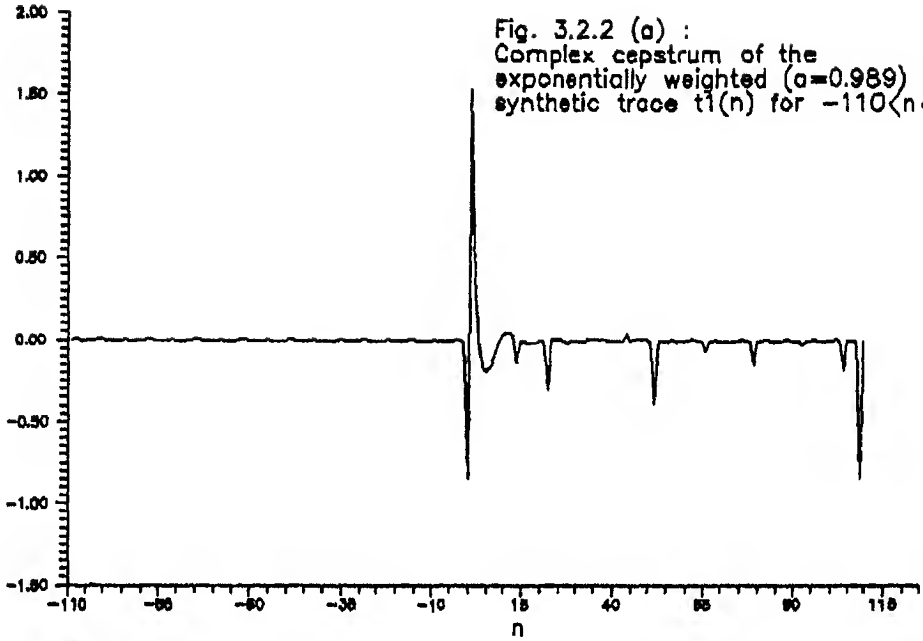


Fig. 3.2.2 (a) :
Complex cepstrum of the exponentially weighted (α=0.989) synthetic trace t1(n) for −110<n<110

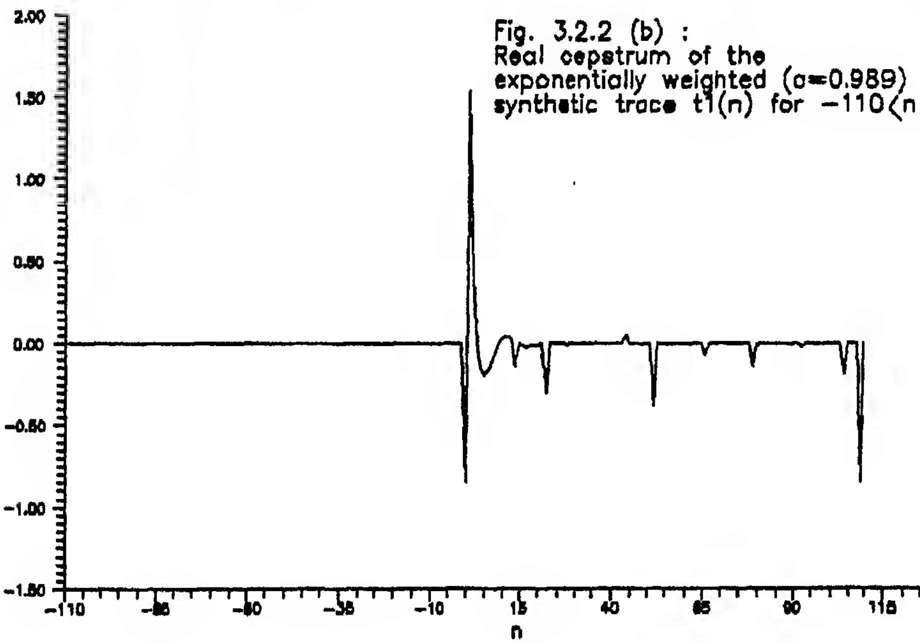


Fig. 3.2.2 (b) :
Real cepstrum of the exponentially weighted (α=0.989) synthetic trace t1(n) for −110<n<110

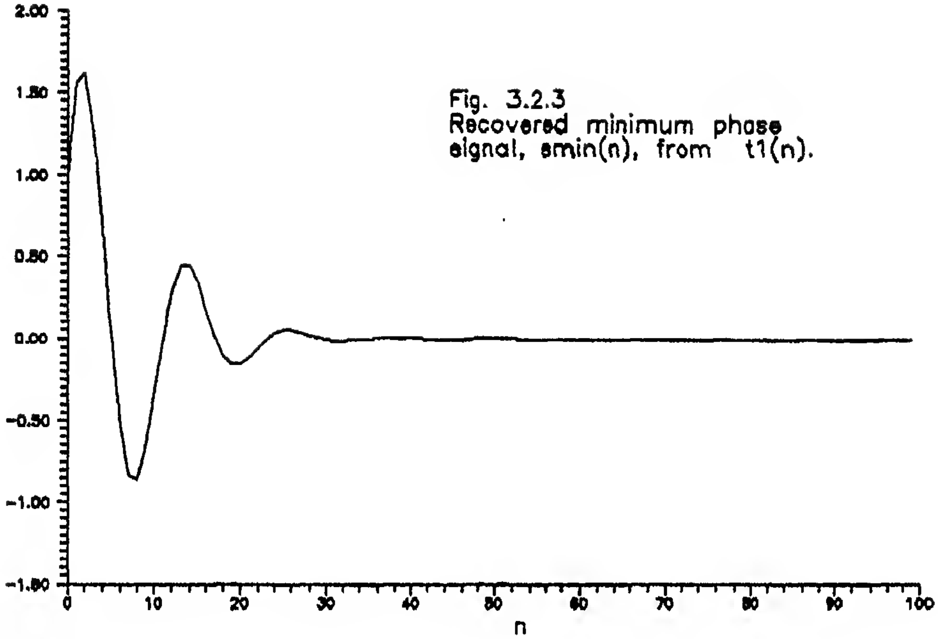


Fig. 3.2.3
Recovered minimum phase signal, smin(n), from t1(n).

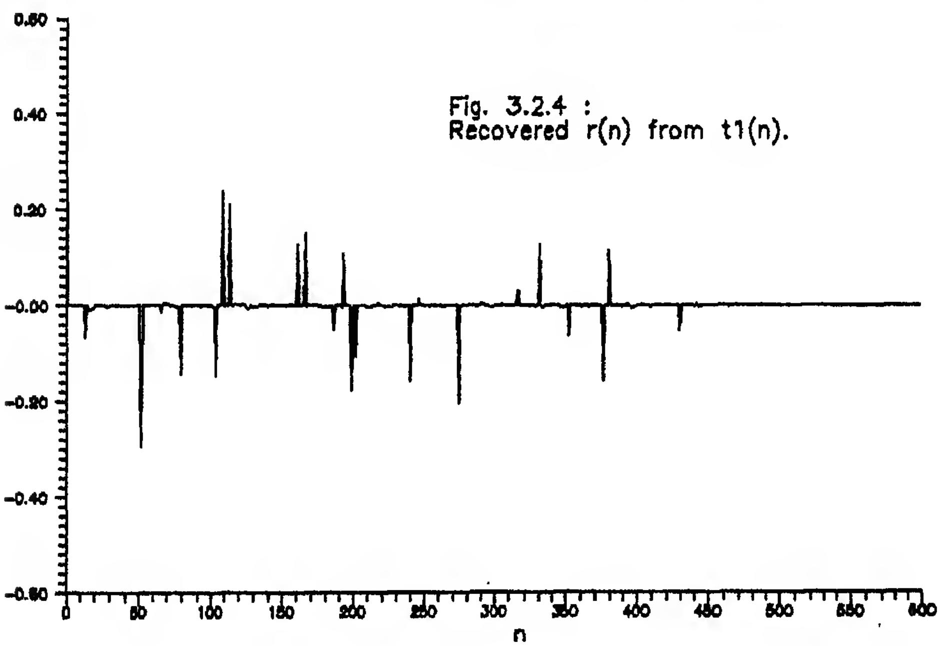


Fig. 3.2.4 :
Recovered r(n) from t1(n).

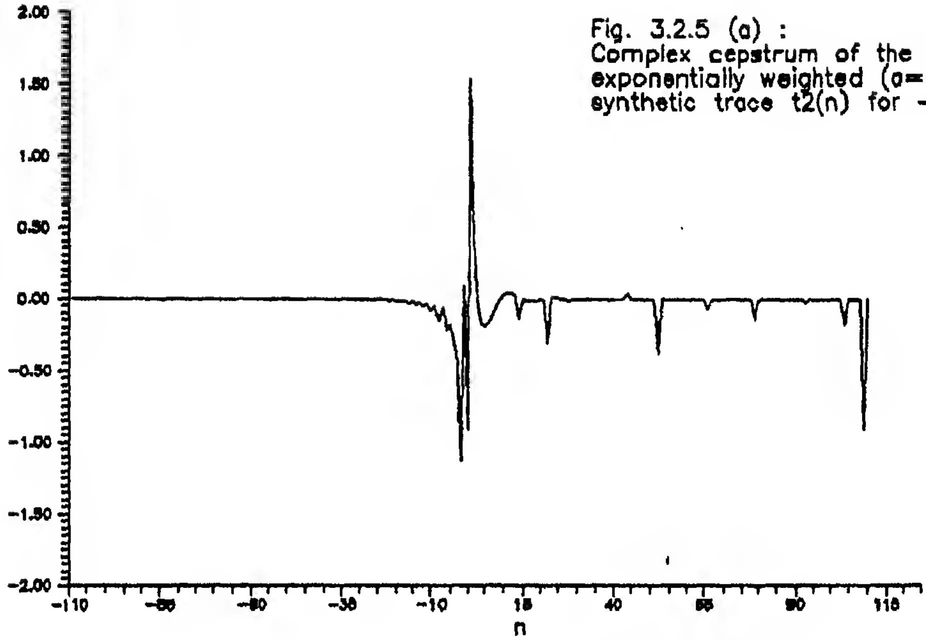


Fig. 3.2.5 (a) :
Complex cepstrum of the exponentially weighted (a=0.989) synthetic trace t2(n) for −110 n 110

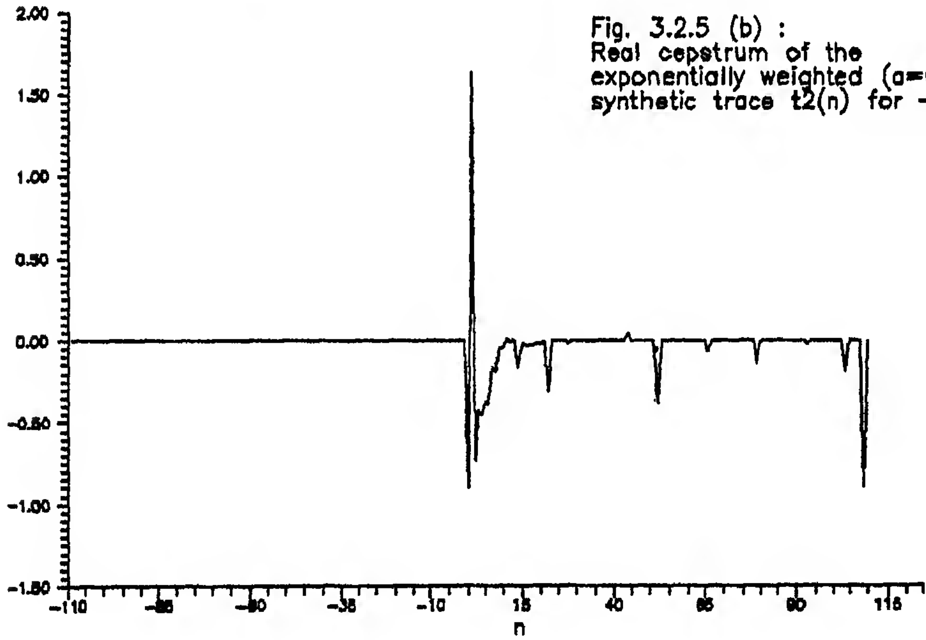


Fig. 3.2.5 (b) :
Real cepstrum of the exponentially weighted (a=0.989) synthetic trace t2(n) for −110 n 110

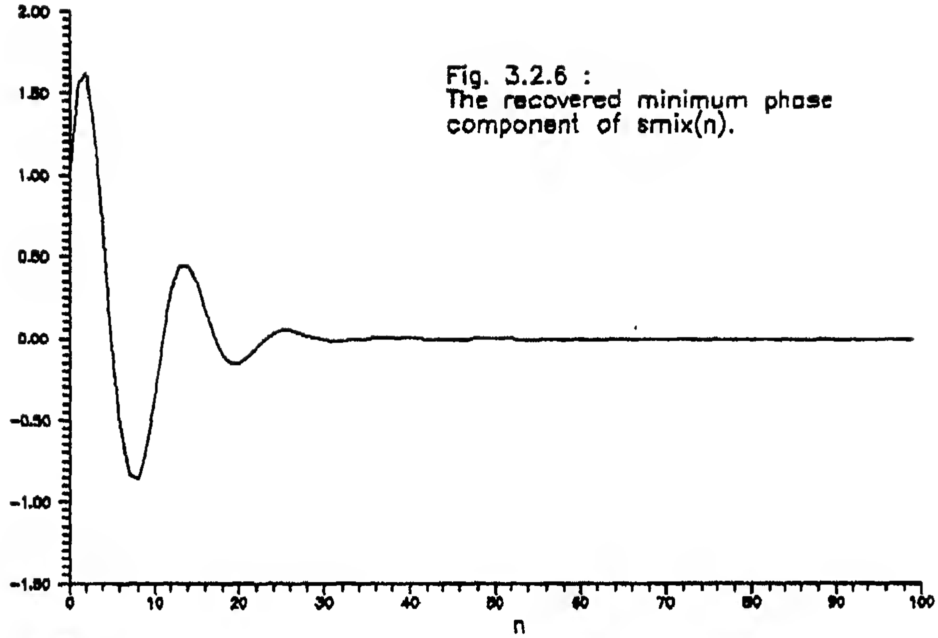


Fig. 3.2.6 :
The recovered minimum phase component of smix(n).

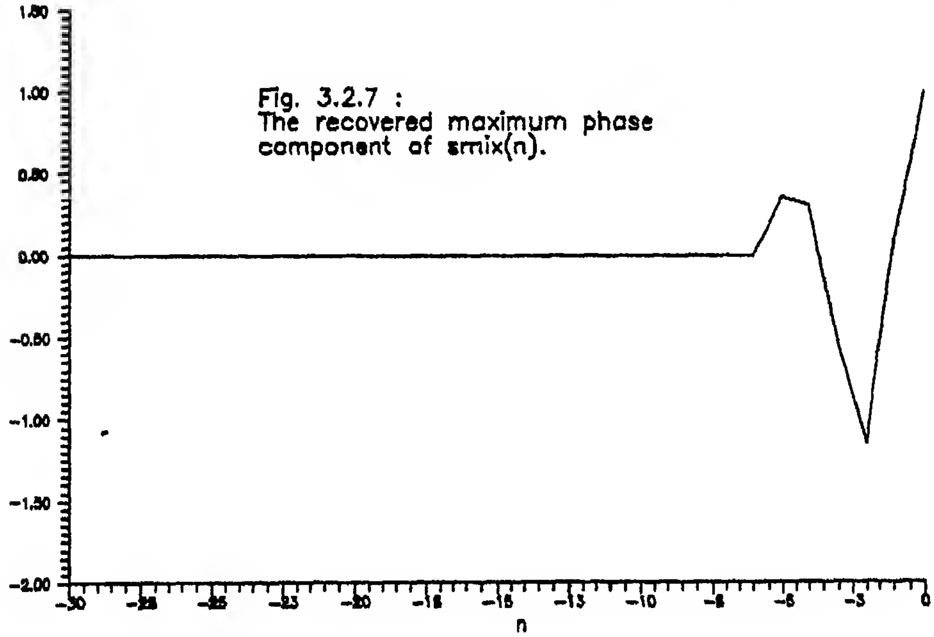


Fig. 3.2.7 :
The recovered maximum phase component of smix(n).

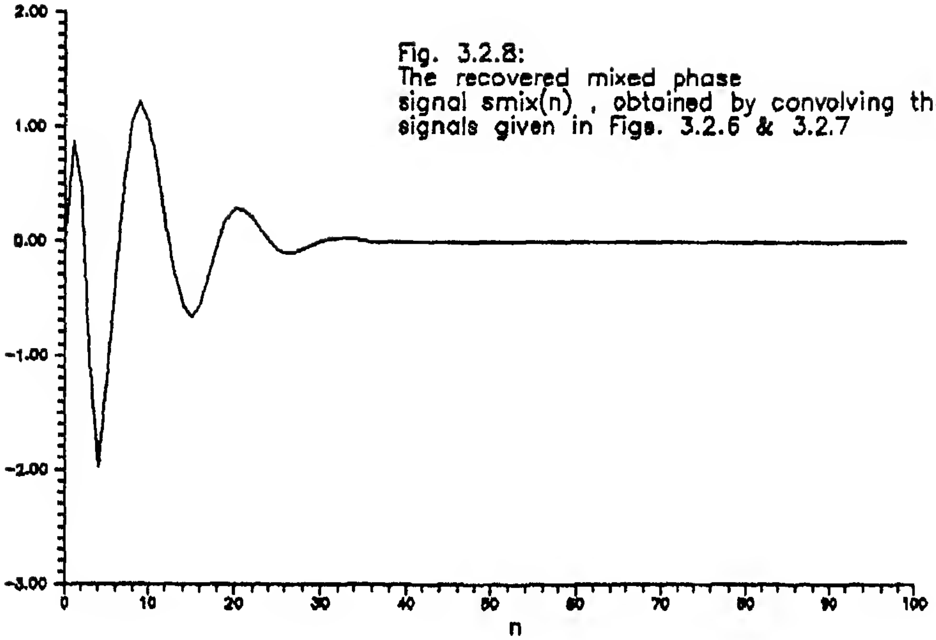


Fig. 3.2.8:
The recovered mixed phase signal smix(n) , obtained by convolving the signals given in Figs. 3.2.6 & 3.2.7

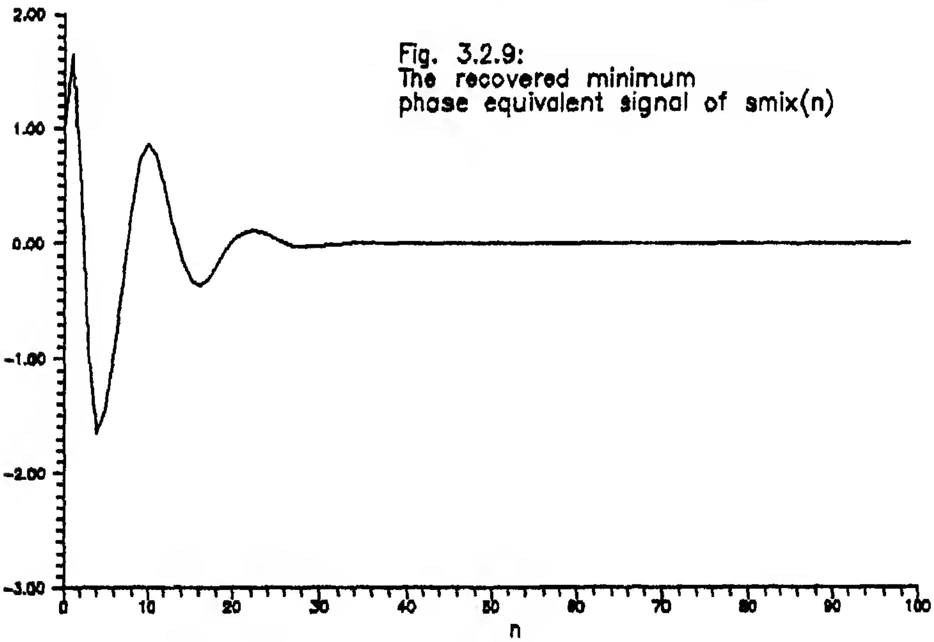


Fig. 3.2.9:
The recovered minimum phase equivalent signal of smix(n)

CHAPTER 4

DECONVOLUTION BY COMBINED PREDICTIVE AND HOMOMORPHIC FILTERING

As pointed out in Chapter-3, the problems associated with the Predictive deconvolution method are : (1). determination of the allpass filter when s(n) is minimum phase , (2). poor results when the sequence r(n) is not a perect random sequence. Also the predictive deconvolution method requires an estimate of the duration of s(n). As discussed in section 3.2 , the Homomorphic deconvolution method in general can not recover r(n). In this section , it is shown that if both Predictive and Homomorphic deconvolution methods are combined , retaining the assumptions made by the former , a significant improvement in the quality of the results can be obtained. Through this method , the problems associated with Predictive and Homomorphic methods when used independently, are solved. In this method , the all-pass component of s(n) and the minimum phase equivalent of r(n) are computed using homomorphic filtering.

4.1 DETERMINATION OF THE ALL-PASS FILTER :

A stable and causal minimum phase sequence has a property that all the zeros of its z-transform lie within the unit circle. But a stable and causal mixed phase signal will have some of its zeros outside the unit circle. This mixed phase sequence can be represented by a cascade of a minimum phase system and an all-pass system having

unit magnitude frequency response. A first order all-pass filter is represented by ,

$$H_{ap}(z) = \frac{(z^{-1} - a)}{(1 - az^{-1})} \;,\; 0<a<1 \;,\quad a \text{ real.} \tag{4.1}$$

Consider a mixed phase system X(z) , with one zero outside the unit circle, at $z = \frac{1}{a}$, ($|a| < 1$, "a" real), and remaining zeros inside the unit circle, i.e.

$$X(z) = X1(z) \,.\, (z^{-1} - a) \tag{4.2}$$

where , X1(z) is minimum phase.

Equation 4.2 can be written as ,

$$X(z) = X1(z) \,.\, (z^{-1} - a) \,.\, \frac{(1 - a.z^{-1})}{(1 - a.z^{-1})}$$

$$= [\; X_1(z) \,.\, (1 - az^{-1}) \;] \,.\, [\; \frac{(z^{-1} - a)}{(1 - az^{-1})} \;]$$

$$= X_{meq}(z) \,.\, H_{ap}(z) \tag{4.3}$$

$X_1(z)$ is called the minimum phase component of X(z) and $X_{meq}(z)$ is called the minimum phase equivalent of X(z). $| H_{ap}(z) | = 1$, for all frequencies.

In eqn. 4.2, x(n) can be considered as the convolution of its minimum phase component $x_1(n)$ and its maximum phase component $x_2(n)$, where, $X_2(z) = z^{-1} - a$. In general, if X(z) has P zeros outside the unit circle, then it can be expressed interms of its minimum and maximum phase components as:

$$X(z) = X_1(z) . X_2(z) \tag{4.4}$$

where,

$$X_2(z) = 1 + \sum_{i=1}^{P} \alpha_i \, z^{-i} \tag{4.5}$$

As in (4.3) X(z) can also be expressed as:

$$X(z) = X_{meq}(z) . H_{ap}(z)$$

where $H_{ap}(z)$ is of the form,

$$H_{ap}(z) = \frac{1 + \beta_1 z^{-1} + \beta_2 z^{-1} + \dots\dots\dots\dots + \beta_P z^{-P}}{\beta_P + \beta_{P-1} z^{-1} + \dots\dots\dots + \beta_1 z^{P-1} + 1} \tag{4.6}$$

Hence it follows that all the zeros of $H_{ap}(z)$ must be the same as those of $X_2(z)$, which implies that $\beta_i = \alpha_i$, $i=1,2,.........P$. Thus we have seen that only the knowledge of $x_2(n)$ is sufficient to determine the corresponding all-pass filter.

Considering the first problem associated with the predictive deconvolution , viz., the determination of the all-pass filter when s(n) is mixed phase , it is sufficient if we know the maximum phase component of s(n) . Homomorphic filtering can be used to get this maximum phase component of s(n) . The random sequence r(n) is made minimum phase after exponential weighting of the synthetic trace and then the complex cepstrum $\hat{t}(n)$ is computed. It is assumed that the exponential weighting does not shift any of the zeros of s(n) lying outside the unit circle to inside the unit circle. The maximum phase component of s(n) whose complex cepstrum is present only at the negetive quefrencies can be recovered by considering the samples of $\hat{t}(n)$ for $n<0$. This recovered signal is just the sequence, $\{ 1, \alpha_1, \alpha_2, ..., \alpha_P \}$. Thus , the required all-pass filter transfer function is determined. The impulse response of this all-pass filter is next computed. The predictive deconvolution requires the deconvolution of this all-pass filter from the recovered r(n). The impulse response of the inverse of this all-pass filter is just the time reversed impulse response of the all-pass filter itself. This is illustrated for a first order all-pass filter :

We have,

$$H_{ap}(z) = \frac{(z^{-1} - a)}{(1 - az^{-1})}$$

$$z.H_{ap}(z) = \frac{(1 - a.z)}{(z - a)}$$

$$[\,z.H_{ap}(z)]^{-1} = \frac{(z-a)}{(1-a.z)}$$

$$= H_{ap}(z^{-1}). \tag{4.7}$$

So the the time reversed impulse response of the all-pass filter is convolved with the the recovered r(n) in Predictive deconvolution method to remove the all-pass filter component from it.

Treitel S. and E.A.Robinson [22] have suggested a different method for the determination of all-pass component. In this method, the mixed phase signal $s_{mix}(n)$ and its minimum phase equivalent, $s_{meq}(n)$ are recovered from the complex and the real cepstra of $a^n.t(n)$ respectively. Next, the $s_{meq}(n)$ is deconvolved from $s_{mix}(n)$ to get the all-pass component. This method requires the accurate choice of the window width. So he has concluded that the determination of the window width in Homomorphic method is equivalent to the determination of the all-pass filter in the Predictive method. On the other hand, the method of determination of the all-pass component from the maximum phase component of $s_{mix}(n)$ does not use windowing. This method assumes that even after exponential weighting all the zeros of the maximum phase component of $s_{mix}(n)$ are still outside the unit circle. This assumption can be made because the zeros of r(n) are much close to the unit as compared to those of s(n). However, we shall illustrate in section—4.4 that, even if few zeros of maximum phase component of $s_{mix}(n)$ lying close to the unit circle are shifted inside the unit circle, the all-pass component so determined does not differ much from the true all-pass component.

4.2 FINDING OUT THE PERIOD OF p(n) :

If the Homomorphic Deconvolution method makes use of the assumption about r(n) to be nearly random, the period of p(n) can be determined. Let r(n) be nearly a random sequence. Hence its autocorrlation function has very small values for $n \neq 0$. Further, the autocorrelation of the periodic sequence p(n), is also a periodic sequence with the same period as p(n). Hence the autocorrelation of p(n) has relatively large amplitudes for the first few periods. Also , the autocorrelation of the convolution of the signals is the convolution of their autocorrelations. The real cepstrum of the autocorrelation of a signal is twice the real cepstrum of the signal. So the real cepstrum of the synthetic trace is computed. This real cepstrum corresponds to a scaled (scaling factor = 1/2) version of the cepstrum of the autocorrelation of the synthetic trace. The large amplitudes at the first few higher quefrencies are mainly due to the p(n) . As the autocorrelation of p(n) . is periodic, the value of the quefrency , n , at which a large spike appears for the first time gives the period of p(n). If the first two cepstral values at n=N , & 2N are suppressed , then the convolutional component p(n) is suppressed at n=N & 2N and its amplitudes at other n will be 1/3 of their original values.

4.3 RESTORING THE MINIMUM PHASE COMPONENT OF r(n):

If the sequence r(n) is not a perfect random sequence then in predictive deconvolution, the prediction error filter computed will be the inverse filter of the convolution of minimum phase equivalent of r(n), the p(n) and the minimum phase equivalent of s(n). Hence the minimum

phase equivalent of the r(n) will also get deconvolved In case of perfectly random r(n), its minimum phase equivalent will be a spike at n=0 and hence its deconvolution does not make any difference If r(n) is not a perfectly random sequence, then its minimum phase equivalent will be nonzero for n>0 and hence its deconvolution makes a considerable difference in the recovered r(n). If this minimum phase equivalent is restored back after predictive deconvolution, then the recovered r(n) will be correct. This restoration is done as follows :

The real cepstrum of the synthetic trace, t(n) is computed The p(n) is suppressed as explained in section 4.2. The synthetic trace is reconstructed. This reconstructed trace is the convolution of the minimum phase equivalents of s(n) and r(n). The minimum phase equivalent of s(n) can be recovered from the low quefrency part of the real cepstrum of the exponentially weighted synthetic trace. This is deconvolved from the above reconstructed trace to get the minimum phase equivalent of r(n).

4.4 ILLUSTRATION :

This illustration is to show the improvements in the results obtained by the Predictive deconvolution, after making use of the Homomorphic Signal Processing.

Consider the synthetic trace, $t_2(1)$.

1. The recovery of r(n) using Predictive deconvolution is explained in section 3.1 and the recovered r(n) is shown in Fig 3.1.4

2. Recovery of minimum phase eqivalent of r(n) :

i. The real cepstrum of $t_2(n)$ is computed (Fig 4.4.1)

0.989 shifts this zero to inside the unit circle. The mixed phase signal $s'_{mix}(n)$, having this zero in addition to those of $s_{mix}(n)$ is shown in Fig. 4.4.7. The all-pass component corresponding to the zero at $z=-1/0.992$ is shown in Fig.4.4.8. Let $s'_{mix}(n)=s'_{meq}(n).h'_{ap}(n)$ as in eqn(4.3). The $h'_{ap}(n)$ is shown in Fig.4.4.9. Then let us try to recover the all-pass component $h'_{ap}(n)$ from $(0.989)^n.s'_{mix}(n)$. The complex cepstrum of $(0.989)^n.s'_{mix}(n)$ is shown in Fig. 4.4.10. The all-pass component computed from the negative quefrencies is shown in Fig. 4.4.11. The all-pass component corresponding to the zero at $z=-1/0.992$ is not present in Fig. 4.4.11. However the recovered (Fig.4.4.11) and the true (Fig.4.4.9) all-pass components are almost the same. The reason for this is that the all-pass component corresponding to a zero close to the unit circle has its samples for $n>0$ much smaller than the sample at $n=0$ (Fig.4.4.8).

4.5 DECONVOLUTION OF SYNTHETIC TRACE FOR DYNAMIC MODEL

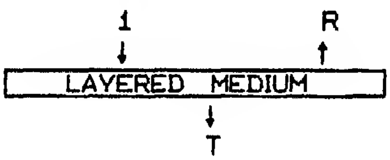
The synthetic trace for the Dynamic Model is :

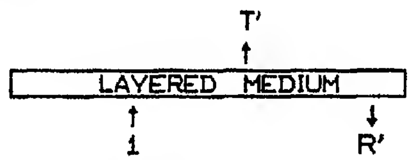
$$t(n) = s(n) * u_1(n)$$

Both Homomorphic and the Predictive filtering are used to recover the reflection coefficients of the layers from t(n). The Homomorphic Signal Processing is used to deconvolve/recover the input s(n) to the layered system. After the deconvolution of the s(n) from the synthetic trace , we get the the unit impulse response $u_1(n)$, of the layered medium . A method called Dynamic Predictive Deconvolution Method, proposed by Robinson [18] , is made use of to recover the reflectin coefficients of each layer from $u_1(n)$. The procedure for the recovery of s(n) is same

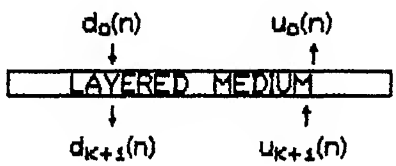
as the one explained in section 3.2. So only the Dynamic Predictive Deconvolution Method is explained in the following section:

The method is based on the assumption that there is no internal loss of energy by absorption within the layers. So the energy from a downgoing unit spike at interface#0 as input is divided between the wave transmitted by the layered system into the last layer and the wave reflected by the layered system above the interface#0. This reflected wave is $u_0(n)$. The reflection (R) and the transmission (T) responses of the layered medium are defined as the responses of the medium to an impulse incident on the surface as shown below :





The relation between arbitrary incoming and outgoing waves , shown below , is expressed as :



From (2.11),
$$\begin{bmatrix} D_{K+1}(z) \\ U_{K+1}(z) \end{bmatrix} = \frac{z^{K/2}}{T^K} \begin{bmatrix} P_K{}^R & Q_K{}^R \\ Q_K & P_K \end{bmatrix} \begin{bmatrix} D_0(z) \\ U_0(z) \end{bmatrix}$$

where , $T^K = \prod_{s=1}^{K} (1-c_s)$, c_i : reflection coefficient of ith interface

From the above figures ,

$$\begin{bmatrix} 1 \\ R \end{bmatrix} = \frac{z^{K/2}}{T^K} \begin{bmatrix} P_K{}^R & Q_K{}^R \\ Q_K & P_K \end{bmatrix} \begin{bmatrix} T \\ 0 \end{bmatrix} \tag{4.8}$$

$$\begin{bmatrix} 0 \\ T' \end{bmatrix} = \frac{z^{K/2}}{T^K} \begin{bmatrix} P_K^R & Q_K^R \\ Q_K & P_K \end{bmatrix} \begin{bmatrix} R' \\ 1 \end{bmatrix} \tag{4.9}$$

The above two equations can be combined as ,

$$\begin{bmatrix} 1 & 0 \\ R & T' \end{bmatrix} = \frac{z^{K/2}}{T^K} \begin{bmatrix} P_K^R & Q_K^R \\ Q_K & P_K \end{bmatrix} \begin{bmatrix} T & R' \\ 0 & 1 \end{bmatrix} \tag{4.10}$$

Solving , for reflection and transmission responses , we get ,

$$R(z) = \frac{Q_K(z)}{P_K(z)} \quad , \quad T(z) = \frac{T^K . z^{-K/2}}{P_K(z)}$$

$$R'(z) = -\frac{Q_K^R(Z)}{P_K(Z)} \quad , \quad T'(z) = \frac{T^K . z^{-K/2}}{P_K(z)}$$

Since on physical grounds , the transmission response $T(z)$ must be the z-transform of a stable and causal sequence , it follows necessarily that $P_K(z)$ must be a minimum phase polynomial.

By the conservation of energy , the input energy is equal to the sum of reflection energy and the transmission energy. The input energy is unity for unit impulse input. The reflection energy is : $R(z)R(z^{-1})$.

Now, $$1 - R(z)R(z^{-1}) = \frac{P_K(z).P_K(z^{-1}) - Q_K(z).Q_K(z^{-1})}{P_K(z).P_K(z^{-1})}$$

It can be shown that ,

$$P_K(z).P_K(z^{-1}) - Q_K(z).Q_K(z^{-1}) = T^K.T_1^K \tag{4.11}$$

where , $$T_1^K = \prod_{s=1}^{K} (1+c_s)$$

So $$1 - R(z).R(z^{-1}) = \frac{T_1^K}{T^K} . T(z).T(z^{-1}) \tag{4.12}$$

$$r(n) = c_0 . \delta(n) + (1 - c_0).u_1(n) \tag{4.13}$$

From eqn 4.12 , the autocorrelation of the transmission response can be computed Since $P_K(z)$ is a minimum phase polynomial , we can compute the prediction-error operator that contracts the transmitted wave to a spike By convolving this prediction error operator with the reflection response, r(n) , we get $q_K(n)$ The coefficients of the polynomial $Q_K(z)$ are approximately equal to the reflection coefficients. But if the exact value of the reflection coeffiocients are required then the following inverse recursion formulas will have to be solved.

$$q_{n-1}(n) = (1-c_n^2)^{-1}.[\ q_n(n) - c_n.p_n(n)\ ] \tag{4.14}$$

$$p_{n-1}(n) = (1-c_n^2)^{-1}.[\ p_n(n) - c_n.q_n(n)\ ] \tag{4.15}$$

4.5.1 ILLUSTRATION OF DYNAMIC PREDICTIVE DECONVLUTION METHOD

The synthetic trace is : $t(n) = s(n) * u_1(n)$.

[I]. Consider the synthetic trace , $t_A(n)$.

1. The minimum phase s(n) is recovered using Homomorphic Signal Processing as explained in section 3.2 . The exponential weighting required to make r(n) minimum phase is found to be equal to 0.991.

2. The s(n) is deconvolved from $t_A(n)$ to get $u_1(n)$. The $u_1(n)$ obtained after the deconvolution of s(n) is shown in **Fig 4.5.1.**

3. The reflection response , r(n) is computed from eqn. 4.13 with c_0=0.9

4. The autocorrelation $\psi(n)$ of r(n) is computed.

5. The autocorrelation of the unit impulse input is also an unit impulse. So the difference between the autocorrelations of the source signal and of the r(n) is an autocorrelation function ϕ , given by ,

$$\phi(0) = 1-\psi(0)$$

$$\phi(n) = -\psi(n) \text{ , for all "n" not equal to 0 .}$$

6. Next the prediction-error operator , $d_0=1, d_1, d_2, \ldots, d_K$ (K=500) is to be computed by solving the normal equations ,

$$\begin{bmatrix} \phi_1 & \phi_0 & \cdots & \phi_{K-1} \\ \phi_2 & \phi_1 & \cdots & \phi_{K-2} \\ \cdots & \cdot & & \cdots \\ \phi_K & \phi_{K-1} & \cdots & \phi_0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \cdots \\ d_K \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix}$$

7. Compute q(n), by convolving the prediction error operator with r(n). This is shown in **Fig 4.5.2**.

8. The exact reflection coefficients are obtained by , the inverse recursion , given by eqns. 4.14 and 4.15. The reflection coefficients so recovered are shown in **Fig 4.5.3**.

[II] . Consider the synthetic trace $t_5(n)$.

In this case , the s(n) is mixed phase . To recover s(n) , then the complex cepstrum of the exponentially weighted (a=0.991) $t_5(n)$ is to be computed. After deconvolving s(n) , the next procedure is same as that for $t_4(n)$, given above.

4.5.2 COMPUTATION OF THE POLYNOMIALL , $Q_K(Z)$, SOLVING MODIFIED YULE-WALKER EQUATIONS

The Dynamic Predictive Deconvolution Method first recovers the polynomial $Q_K(z)$ from the reflection response , r(n) , and then using reverse recursion , computes the reflection coefficients. But $Q_K(z)$ can be computed by the standard system identification problem also.

The z-transform of r(n) , is given by ,

$R(z) = \frac{Q_K(z)}{P_K(z)}$, both the polynomials $Q_K(z)$ and $P_K(z)$ are polynomials of order equal to K.

First , the AR-polynomial $P_K(z)$ (K=453) is computed from r(n) by solving the modified Yule-Walker equations given by ,

$$\begin{bmatrix} r(K) & r(K-1) & \ldots & r(1) \\ r(K+1) & r(K) & \ldots & r(2) \\ . & & \ldots & \ldots \\ r(2K-1) & r(2K-2) & \ldots & r(K) \end{bmatrix} \begin{bmatrix} p(1) \\ p(2) \\ \ldots \\ p(K) \end{bmatrix} = - \begin{bmatrix} r(K+1) \\ r(K+2) \\ \ldots \\ r(2K) \end{bmatrix} \quad (4.16)$$

where , p(n) are the autoregressive coefficients of the polynomial , $P_K(z)$. p(0)=1. The above modified equations are called *extended* , *modified* , *or higher order Yule - Walker equations*. These equations are solved by modified Levinson's recursive algorithm , which is used in the case of the matrix having Toeplitz structure. The number of multiplications needed for this modified Levinson's recursion is of the order of K^2 , but is about two times that required for the inversion of the Toeplitz matrix of order K.

The MA - coefficients q(n) of the polynomial $Q_K(z)$ are computed by convolving p(n) with the reflection response , r(n). The MA-coefficients computed from r(n) (obtained in step-3 of section 4.5.1) using this method is shown in **Fig 4.5.4**. The disadvantage of this method is that the order of the MA-polynomial , $Q_K(z)$, must be known beforehand. If a wrong order is used , the computed MA-coefficients will not be correct. Also , this method gives only the MA-coefficients. Again the inverse recursion equations 3.43 are to be used to find the exact reflection coefficients.

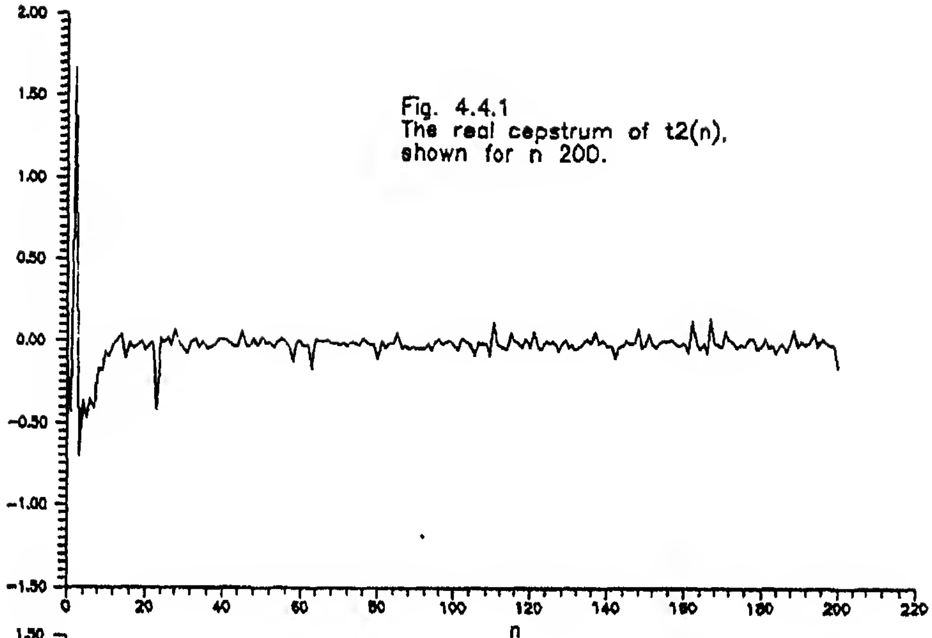


Fig. 4.4.1
The real cepstrum of t2(n),
shown for n 200.

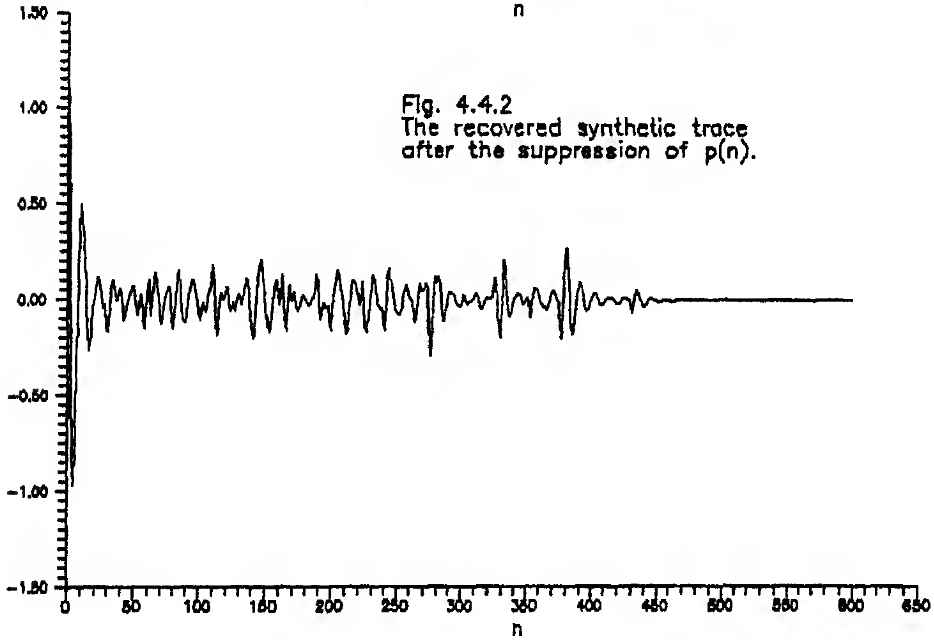


Fig. 4.4.2
The recovered synthetic trace
after the suppression of p(n).

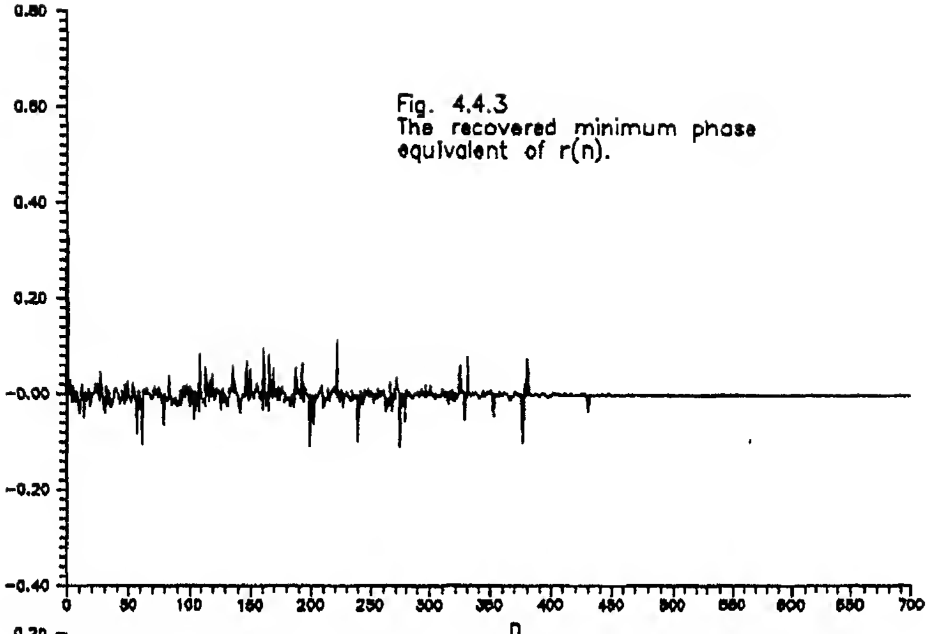


Fig. 4.4.3
The recovered minimum phase equivalent of r(n).

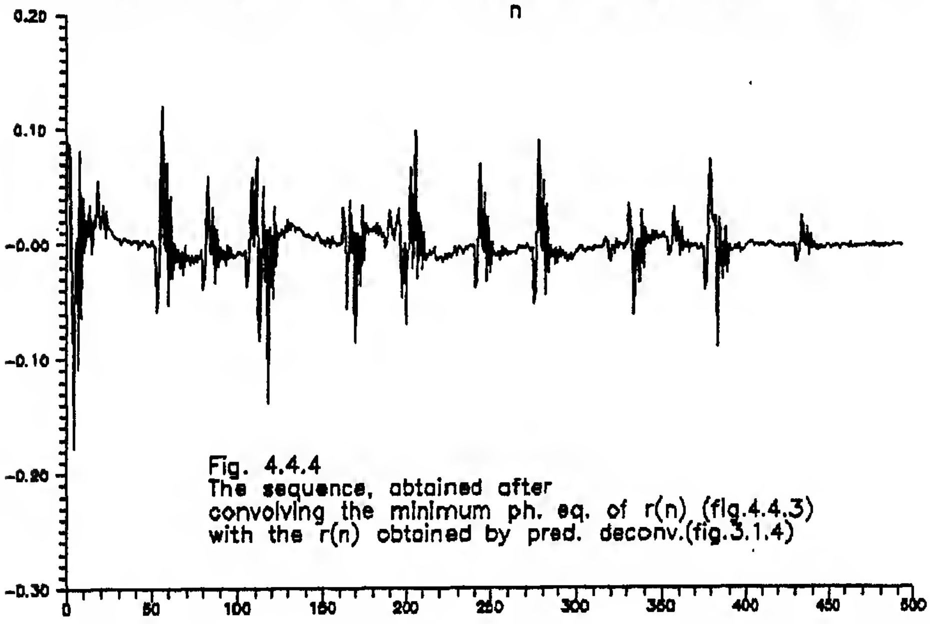


Fig. 4.4.4
The sequence, obtained after convolving the minimum ph. eq. of r(n) (fig.4.4.3) with the r(n) obtained by pred. deconv.(fig.3.1.4)

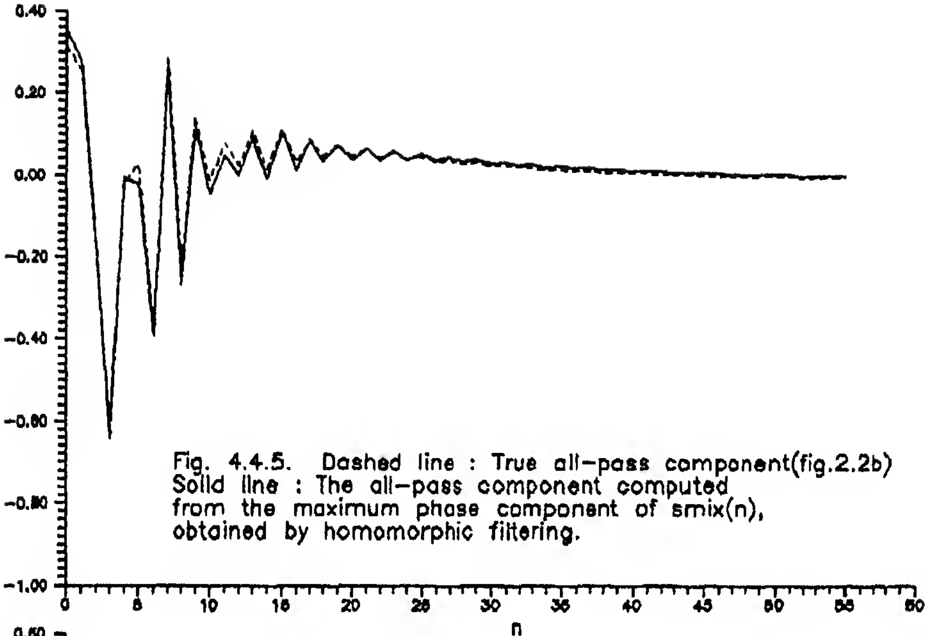


Fig. 4.4.5. Dashed line : True all-pass component(fig.2.2b)
Solid line : The all-pass component computed
from the maximum phase component of smix(n),
obtained by homomorphic filtering.

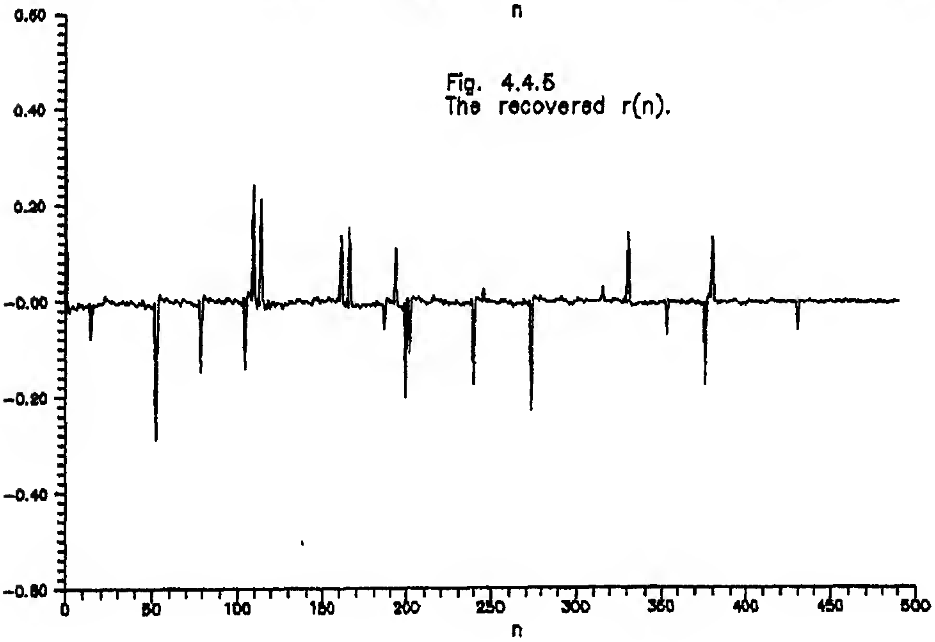


Fig. 4.4.5
The recovered r(n).

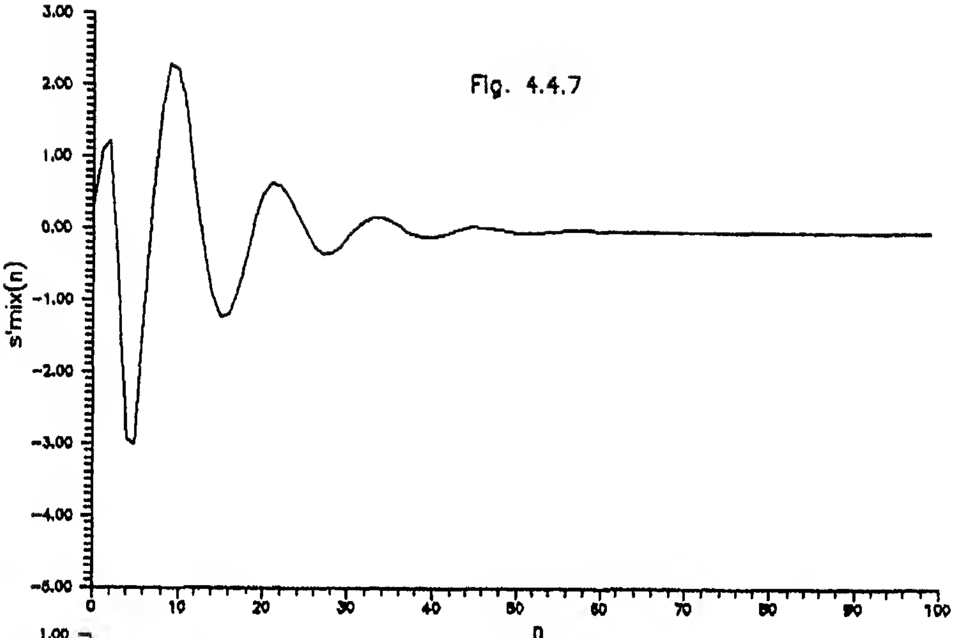


Fig. 4.4.7

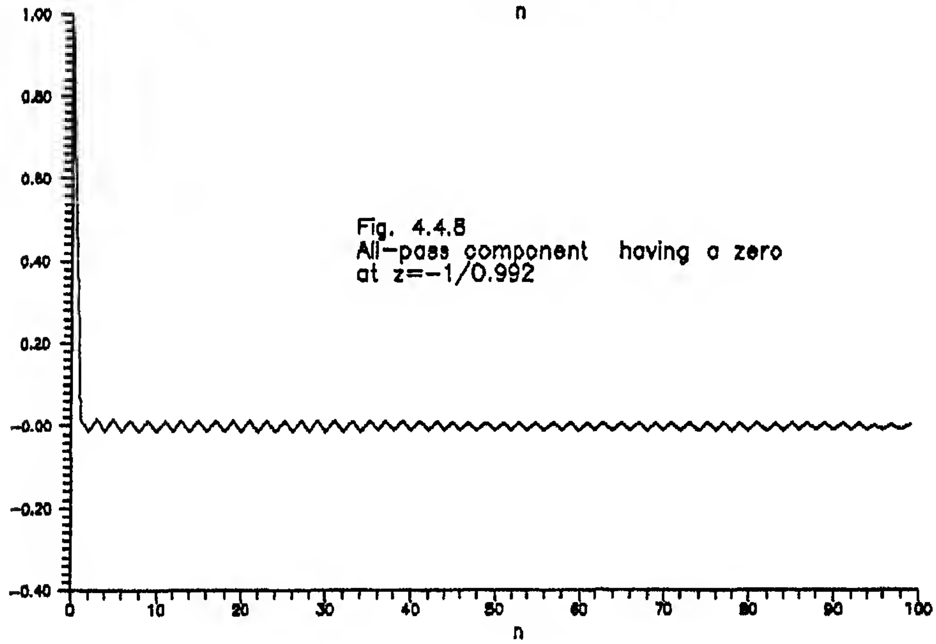


Fig. 4.4.8
All-pass component having a zero at z=-1/0.992

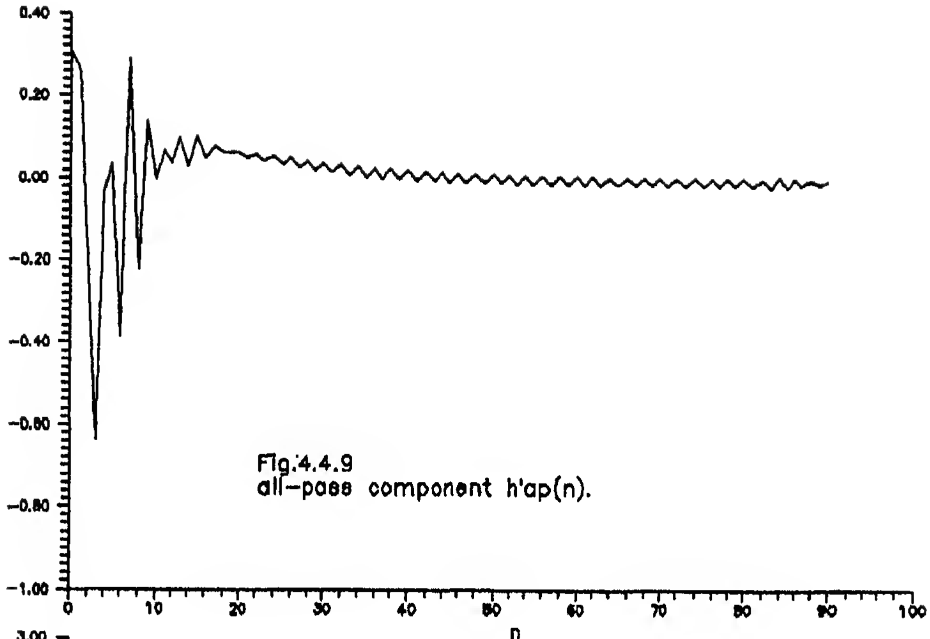


Fig.4.4.9
all-pass component h'ap(n).

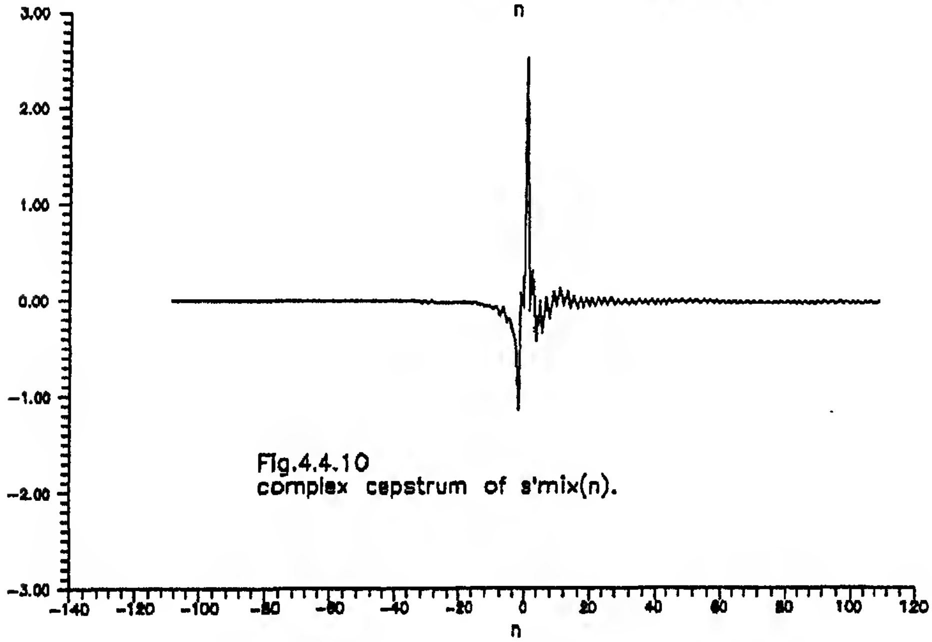


Fig.4.4.10
complex cepstrum of s'mix(n).

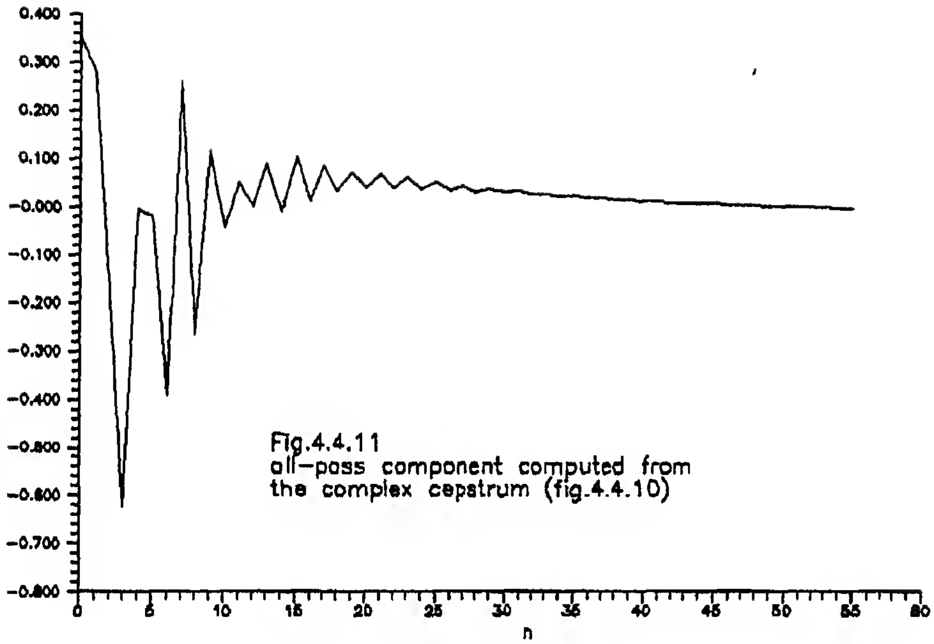


Fig.4.4.11
all-pass component computed from
the complex cepstrum (fig.4.4.10)

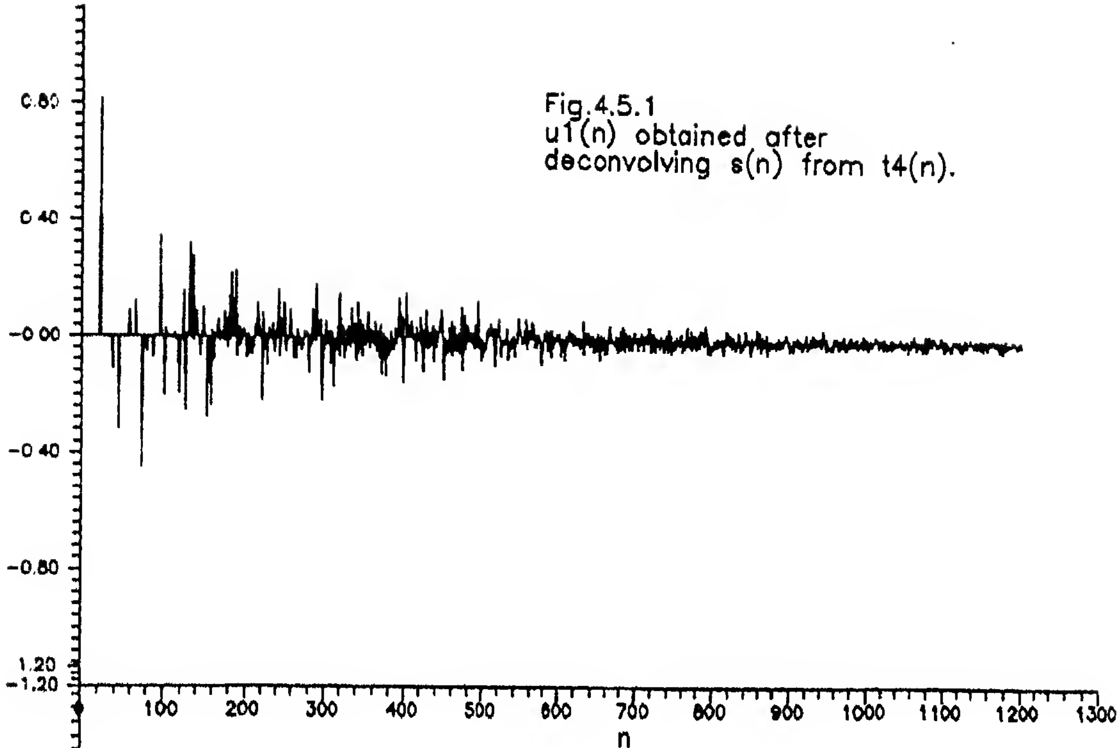


Fig.4.5.1
u1(n) obtained after
deconvolving s(n) from t4(n).

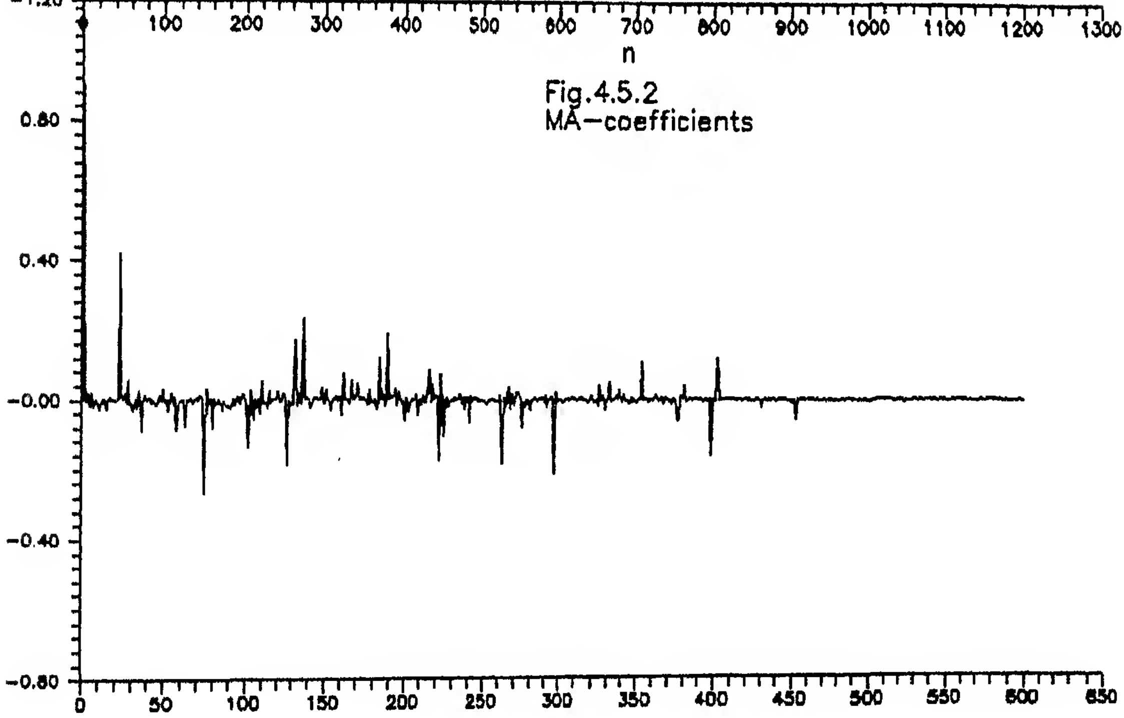


Fig.4.5.2
MA-coefficients

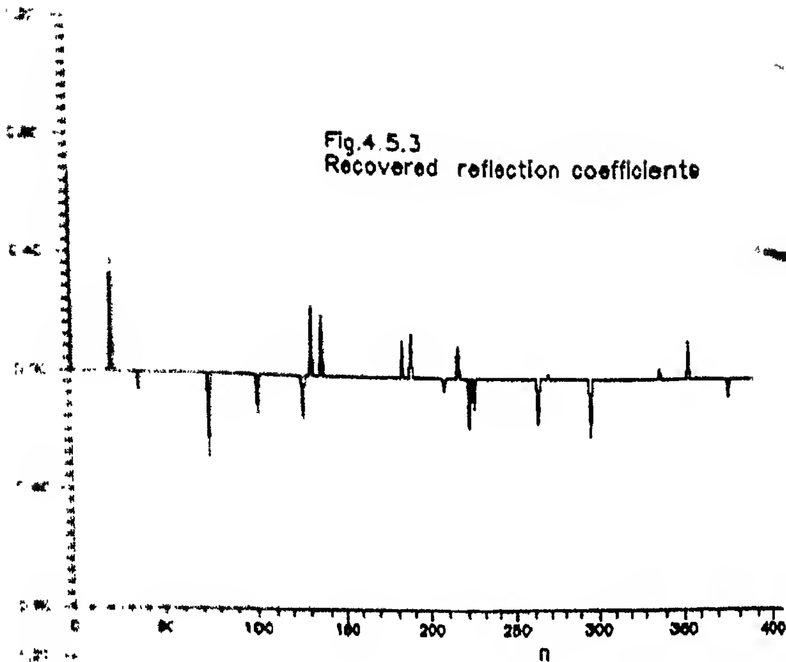


Fig.4.5.3
Recovered reflection coefficients

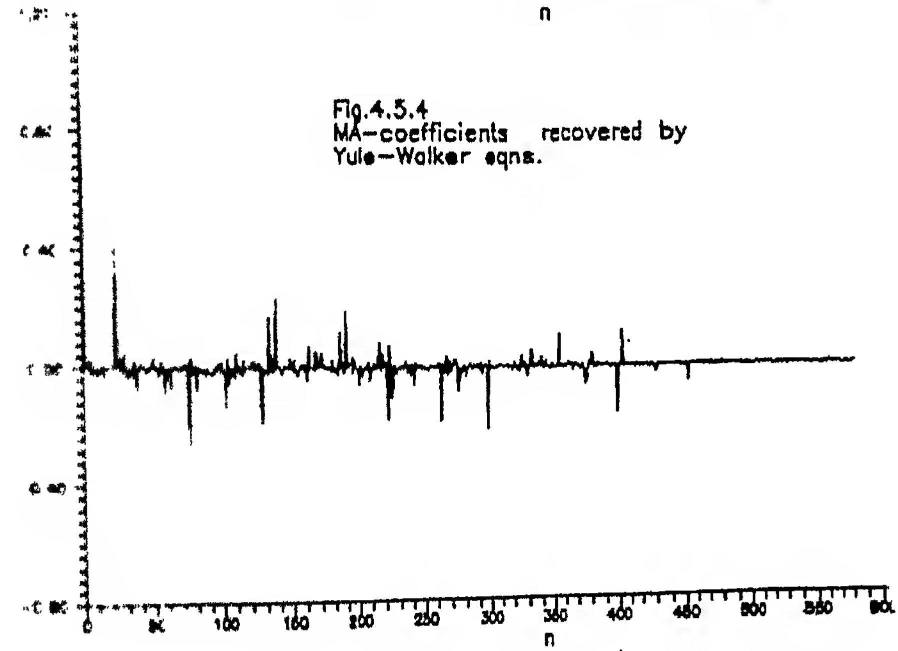


Fig.4.5.4
MA-coefficients recovered by
Yule-Walker eqns.

CHAPTER 5

THE INFLUENCE OF RANDOM NOISE ON HOMOMORPHIC FILTERING

Additive noise plays a critical role in homomorphic deconvolution to such an extent that the method becomes unreliable whenever the spectral amlitudes of the signal are very small over certain frequency bands and even a small amount of noise is present [8]. That part of the noise which does not overlap the signal in the frequency spectrum can be eliminated by bandpass filtering. However the noise often overlaps the signal.

The synthetic trace with additive noise is given by,

$$x(n) = t(n) + \eta(n) \tag{5.1}$$

where, $t(n) = s(n)*r(n)*p(n)$ and $\eta(n)$ is the additive noise.

$$X(\omega) = T(\omega) + \eta(\omega)$$

$$\begin{aligned} \log [\ X(\omega)\ ] &= \log [\ T(\omega) + \eta(\omega)\ ] \\ &= \log [\ T(\omega)\ ] + \log [\ 1 + \frac{\eta(\omega)}{T(\omega)}\ ] \\ &\simeq \log [\ T(\omega)\ ] + \frac{\eta(\omega)}{T(\omega)} \end{aligned} \tag{5.2}$$

where it is assumed that, $\left| \frac{\eta(\omega)}{T(\omega)} \right| \ll 1$ (5.3)

Phase of $X(\omega)$, the imaginary part of (5.2) is equal to,

$$\phi_t(\omega) + \left| \frac{\eta(\omega)}{T(\omega)} \right| \sin [\ \phi_n(\omega) - \phi_t(\omega)\ ] \tag{5.4}$$

where, ϕ_t, ϕ_n are the phases of the signal i.e. synthetic trace and the noise respectively.

Thus it is clear from eqn.(5.4) that the phase of the trace becomes unpredictable and hence the phase unwrapping required in the

computation of complex cepstrum will be unreliable due to the additive noise So the influence of random noise is quite critical in the computation of the complex cepstrum. In addition to this , noise is amplified during the computation of the cepstrum. The following factors influence the noise amplification [3] :

1 The nonlinear logarithmic operation exaggerates small components of the spectrum in the complex spectrum.

2. Due to the fact that the noise is additive , it influences the complex cepstrum in a rather complicated way.

3. The addition of random noise complicates primarily the phase spectrum , which has a strong influence on the complex cepstrum. However it is found that [3] the noise infuene is more severe on the high quefrency part of the cepstrum as compared to that around zero quefrencies.

5.1 ILLUSTRATION :

The influence of noise on homomorphic filtering is demonstrated for SNR=3, 10 and 40.

1. White guassian noise is added to the synthetic trace $t_1(n)$. The noisy trace for SNR = 40, 10 and 3 are shown in **Figs. 5.1.1, 5.1.2 , 5.1.3** respectively.

2. The real and the complex cepstra for the exponentially weighted (a=0.989) noisy trace, SNR=40 are shown in **Fig. 5.1.4, 5.1.5**. From fig. 5.1.4 it is clear that the complex cepstrum is influenced by the noise to a great extent even for high SNR (SNR=40). But we see that the low quefrency part of the real cepstrum is not very much affected by noise.

3. **Figs. 5.1.6, 5.1.7** & **5.1.8** show the recovered $s_{min}(n)$ from the three noisy

3 Figs. 5.1.6, 5.1.7 & 5.1.8 show the recovered $s_{min}(n)$ from the three noisy traces

It is observed that s(n) can not be recovered from the complex cepstrum for all the three cases, wheras the recovery of s(n) from real cepstrum is possible for all the three cases. However as the SNR decreases, the recovery also becomes poor.

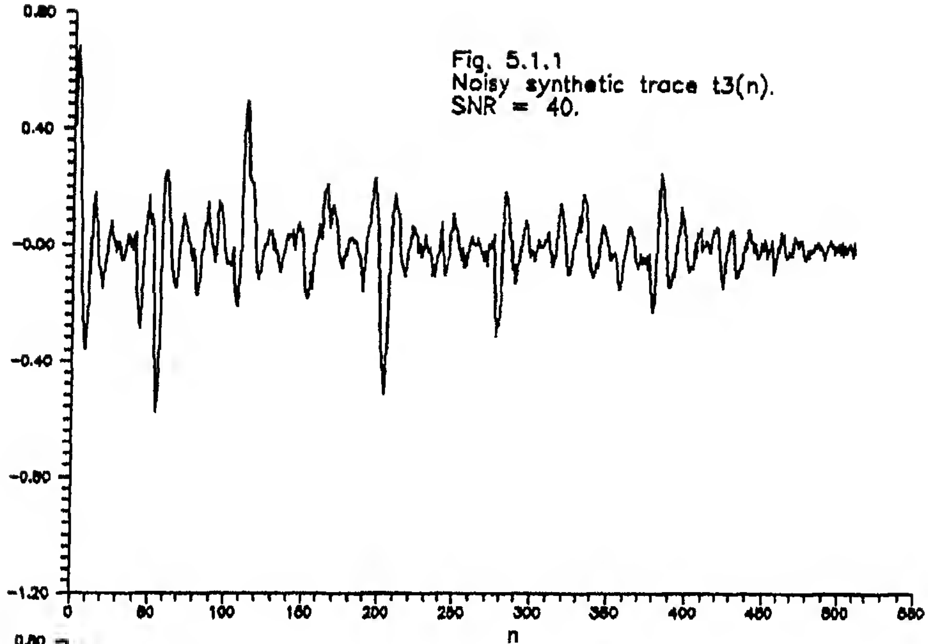


Fig. 5.1.1
Noisy synthetic trace t3(n).
SNR = 40.

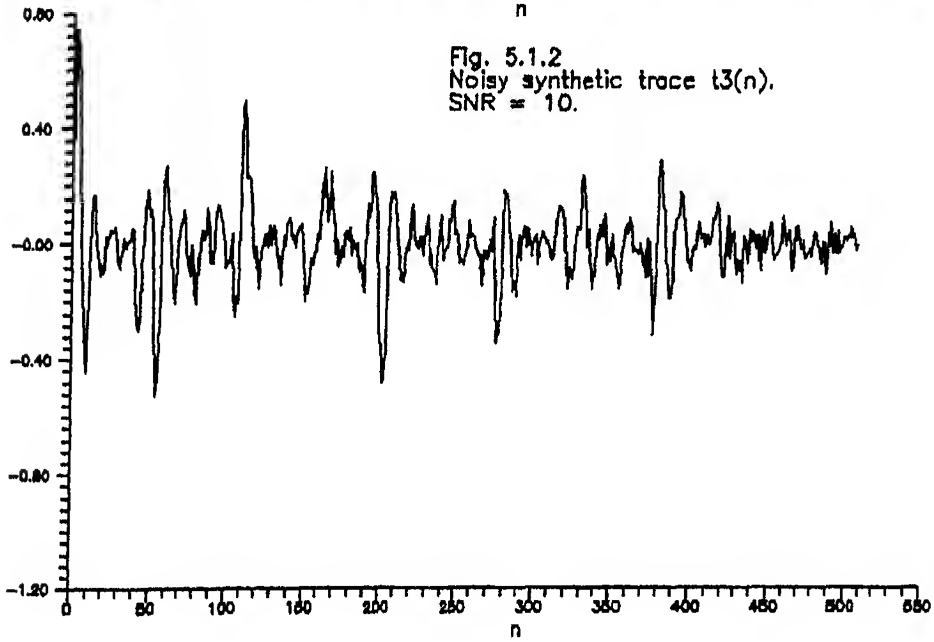


Fig. 5.1.2
Noisy synthetic trace t3(n).
SNR = 10.

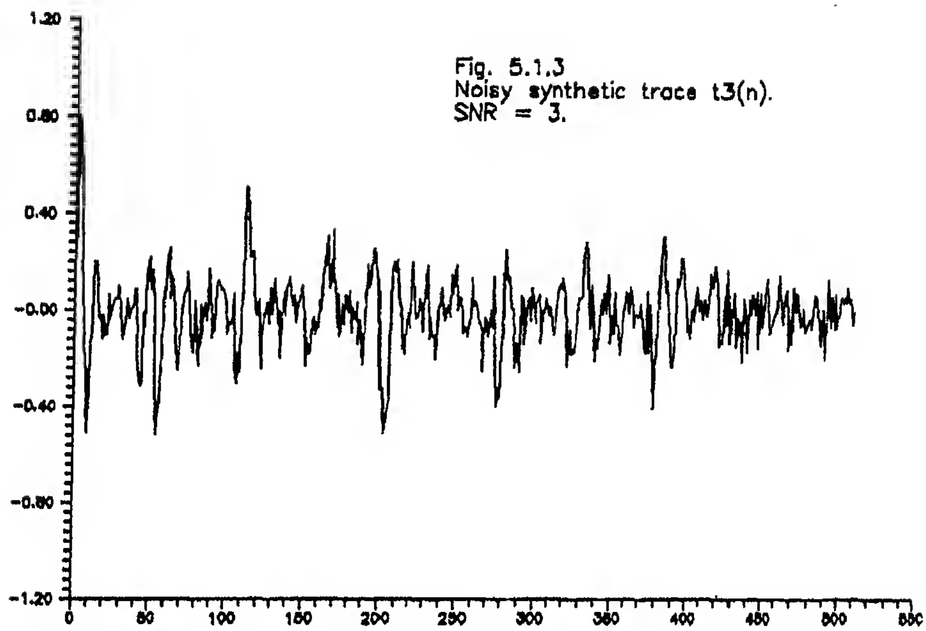


Fig. 5.1.3
Noisy synthetic trace t3(n).
SNR = 3.

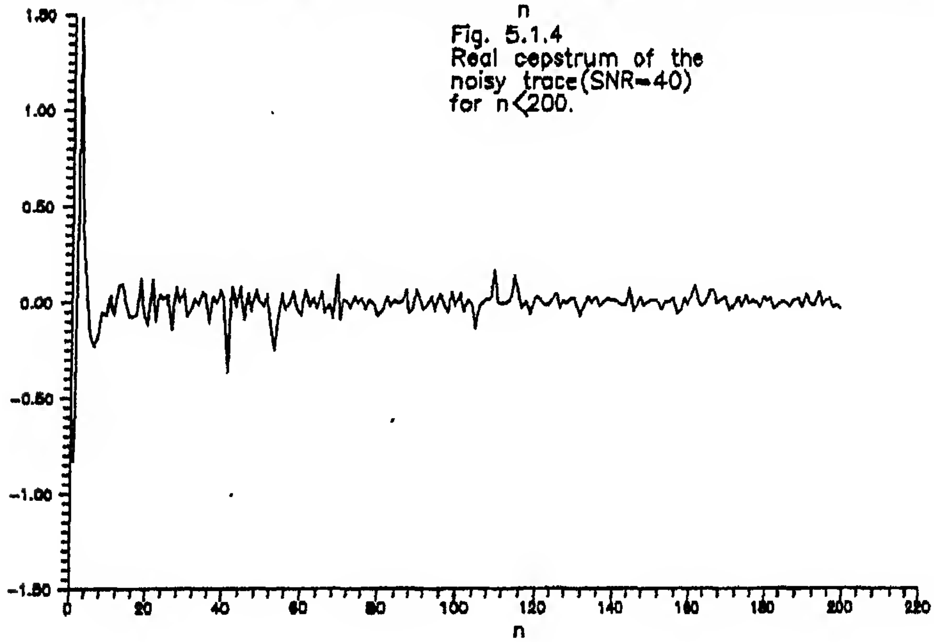


Fig. 5.1.4
Real cepstrum of the
noisy trace(SNR=40)
for n<200.

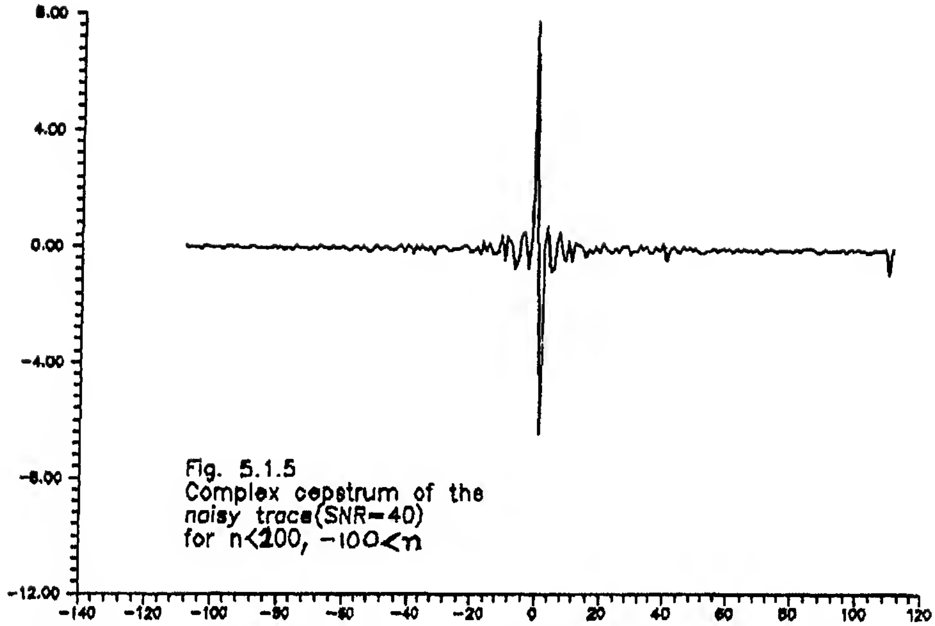


Fig. 5.1.5
Complex cepstrum of the
noisy trace(SNR=40)
for $n<200$, $-100<n$

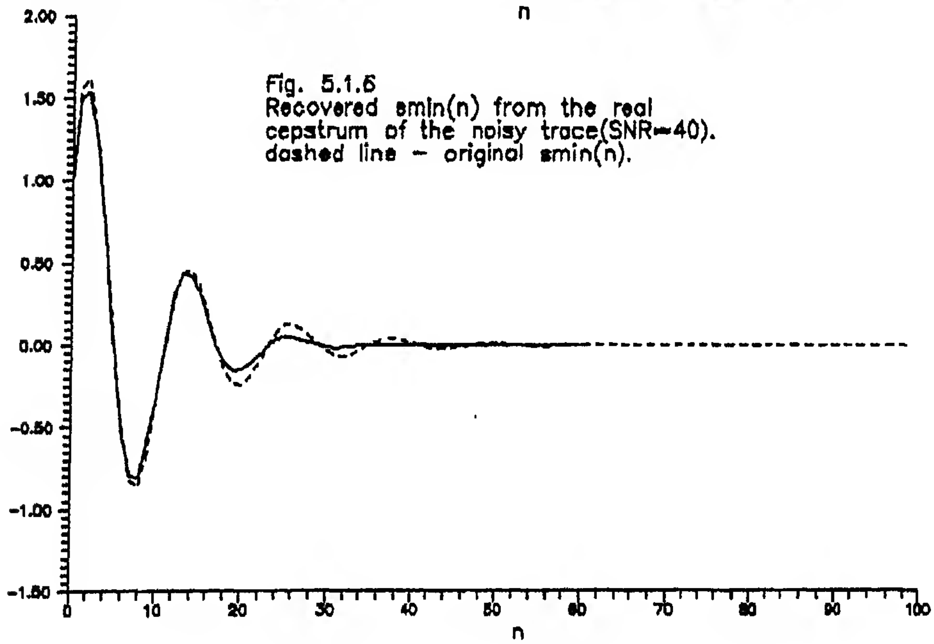


Fig. 5.1.6
Recovered smin(n) from the real
cepstrum of the noisy trace(SNR=40).
dashed line - original smin(n).

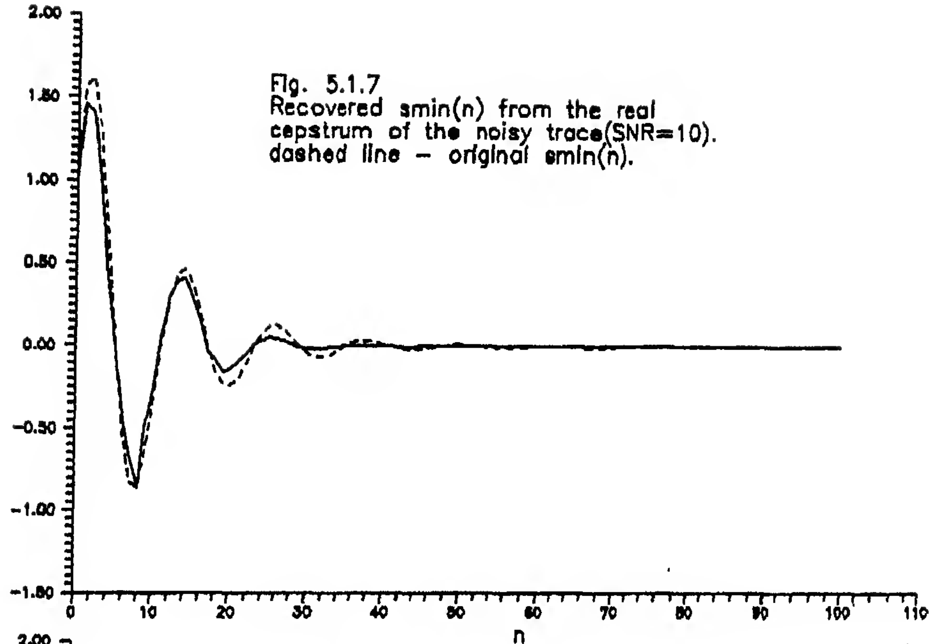


Fig. 5.1.7
Recovered smin(n) from the real cepstrum of the noisy trace(SNR=10).
dashed line – original smin(n).

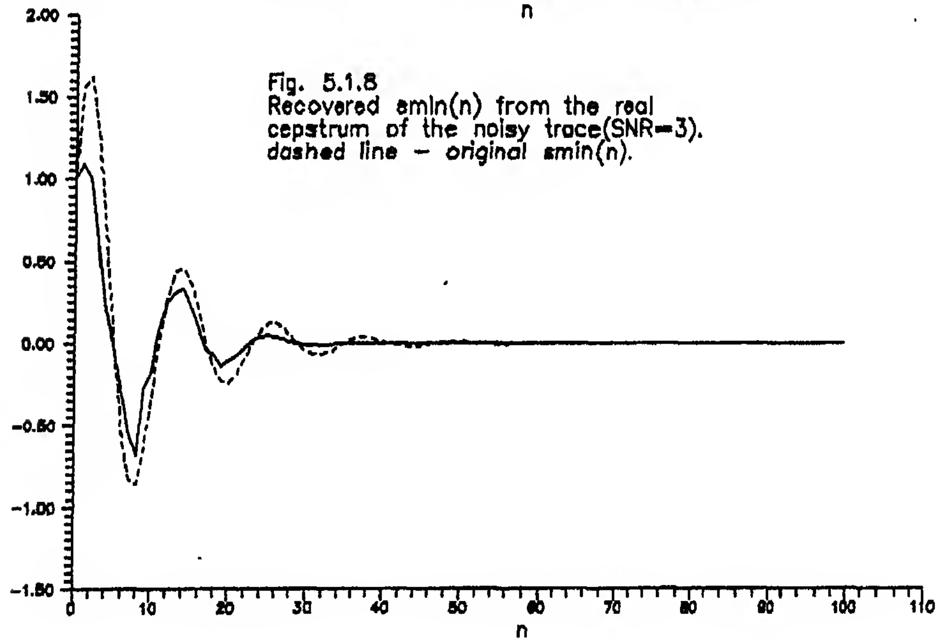


Fig. 5.1.8
Recovered smin(n) from the real cepstrum of the noisy trace(SNR=3).
dashed line – original smin(n).

CHAPTER - 6

CONCLUSION

The main purpose of the present work is to try to solve the problems associated with the predictive deconvolution method. Synthetic traces are computed and then used to study the problems associated with the predictive deconvolution and to show the improvement in the results by combining predictive and homomorphic filtering.

In random model, the predictive deconvolution method assumes s(n) to be minimum phase and r(n) to be an uncorrelated sequence. If s(n) is a mixed phase sequence, then the recovered r(n) will contain an all-pass component. If r(n) is not a perfect white sequence, then its minimum phase equivalent sequence also gets deconvolved. To tackle these problems, homomorphic filtering is made use of along with the predictive filtering. The minimum phase equivalent of r(n) and the all-pass component of the mixed phase s(n) are computed by homnomorphic filtering. Then the all-pass component is deconvolved from the recovered r(n), followed by the restoration of the minimum phase equivalent of r(n).

The success of the homomorphic filtering depends upon the exponential weighting required to make r(n) minimum phase. A trial and error method is proposed in this thesis. If the exponential weighting does not shift any of the zeros of $s_{mix}(n)$ lying outside the unit circle to inside the unit circle, then the maximum phase component of the exponentially weighted trace is the maximum phase component of $s_{mix}(n)$ itself. The all-pass component is computed from the maximum phase

component of the $s_{mix}(n)$ This has an advantage that there is no need to choose the width of the cepstral window. On the other hand , if the exponentially weighting shifts some of the zeros of $s_{mix}(n)$ to inside the unit circle , then the all-pass component corresponding to those zeros can not be computed by homomorphic filtering.

In dynamic model , the dynamic predictive deconvolution (DPD) method can be applied to unit impulse response of the layered medium. So first homomorphic filtring is used to deconvolve s(n) from the trace. Then the DPD is used to recover the reflection coefficients of the layers.

The effect of additive noise on Homomorphic deconvolution is studied. The detailed effects are not well understood [8]. The noise tends to influence the long time (i.e. high quefrency) portion of the cepstrum[23]. In this thesis this aspect has been observed in case of the noisy trace. Buttkus [3] has suggested that by high pass filtering the continous phase spectrum, the influence of the random noise at the deconvolution process can be made reasonably small. He also mentions that though the experimental results confirms this, it is not known whether it is generally true. Thus , the problem , homomorphic deconvolution in presence of random noise, remains to be resolved [8]. Further, better procedure for the determination of the exponential weighting needs to be evolved.

APPENDIX

HOMOMORPHIC SYSTEMS FOR CONVOLUTION

Homomorphic systems are a class of nonlinear systems which satisfy a generalized principle of superposition. Any nonlinear system that satisfies the convolutional superposition property illustrated in **Fig. A-1** is called a *homomorphic system for convolution*. The procedure for homomorphic filtering is shown in **Fig. A-2** and includes the following steps :

[1] Computation of the complex spectrum of $x(n)$:

The signal $x(n)$ is the convolution of two signals , $s(n)$ and $r(n)$. So the DFT of $x(n)$ is ,

$$X(k) = S(k)\, R(k) = |X(k)|.e^{j\theta(k)} .$$

[2]. LOG operation :

The computation of the natural logarithm of the complex spectrum gives the additive superposition of the individual parts :

$$\log X(k) = \log S(k) + \log R(k) = \log|X(k)| + j\theta(k)$$

[3]. Computation of the complex cepstrum :

The computation of the IDFT of the logarithm of the complex spectrum of $x(n)$ gives the complex cepstrum of $x(n)$.

$$\mathrm{IDFT}[\log X(k)] = \hat{x}(n) = \hat{s}(n) + \hat{r}(n).$$

Now the new "time domain" is called the *quefrency domain*. The linear filtering of the complex cepstrum to suppress the undesired parts is called *liftering*. The reverse run through the the first three operations as shown in **Fig. A-2** provides the final result of the homomorphic filtering in time domain.

PROPERTIES OF COMPLEX CEPSTRUM :

[1] The complex cepstrum of a real sequence is also a real sequence.

[2] The complex cepstrum decays at least as fast as $\frac{1}{n}$.

[3] If $x(n)$ is of finite duration $\hat{x}(n)$ is nevertheless have infinite duration.

[4] The complex cepstrum of a minimum phase sequence is zero for $n<0$.

[5]. The complex cepstrum of a maximum phase sequence is zero for $n>0$.

[6]. The complex cepstrum of a minimum phase sequence satisfies

$$\hat{x}_{min}(n) = x_{min}(n) - \sum_{k=1}^{n-1} (\frac{k}{n}) \hat{x}_{min}(n) \, x_{min}(n-k)$$

[7]. The complex cepstrum of a maximum phase sequence satisfies

$$\hat{x}_{max}(n) = x_{max}(n) - \sum_{k=n+1}^{-1} (\frac{k}{n}) \hat{x}_{max}(n) \, x_{max}(n-k)$$

[8]. The complex cepstrum of a pulse, whose spectrum is smooth tends to be concentrated around low quefrencies.

[9]. Let $r(n)$ be a periodic sequence with period $T>1$, defined by

$$r(n)= \sum_{k=0}^{\infty} \alpha_k . \delta(n-kT)$$

where α_i's are the impulse amlitudes.

Then the complex cepstrum $\hat{r}(n)$ of $r(n)$ is also a periodic impulse train with the same period T.

[10]. The removal of the first n complex cepstrum contributions of a sequence $p(n)$ defined by

$$p(n)=\sum_{k=0}^{\infty}(-1)^k a^k \delta(n-kT) \; ,$$

eliminates the first n time domain amplitudes of $r(n)$ and reduces its remaining aplitudes to at most $\frac{1}{n+1}$ of their original amplitudes.

[11]. Consider a minimum phase impulse train $r(n)$ for which the duration between the first two nonzero samples is N, so that

$$n_2-n_1=N \; , \quad n_k \text{ arbitrary for } k>2$$

Then the complex cepstrum of such a sequence is zero for $0<n<N$. In general, $\hat{r}(n)$ is nonzero only at quefrencies, 0, n_2-n_1, n_3-n_1, ..., n_m-n_1, as well as at all positive linear combinations of these times.

[12] Given a minimum delay sequence $x(n)$ we form the reverse sequence $x(-n)$ by folding $x(n)$ about the origin. $\hat{x}(n)$ is the real cepstrum of $x(n)$. Then the real cepstrum of $x(-n) = \hat{x}(-n)$

[13] Given a minimum delay sequence $b(n)$ we define the inverse sequence, $b^{-1}(n)$ by

$$b(n) * b^{-1}(n) = \delta(n).$$

Then the real cepstrum of $b^{-1}(n)$ is equal to $-\hat{b}(n)$.

[14] Real cepstrum of $b^{-1}(-n)$ is equal to $-\hat{b}(-n)$.

[15] A minimum phase sequence can be recovered from its real cepsrum (Fig. A-3).

[16] The real cepstrum of the autocorrelation of a sequence is twice the real cepstrum of the sequence.

[17] With $x(n)$ expressed as the convolution of its minimum phase and maximum phase components, denoted as $x_{min}(n)$ and $x_{max}(n)$, respectively,

$$\hat{x}(n) = \hat{x}_{min}(n) + \hat{x}_{max}(n)$$

[18]. If $x(n)$ has a rational z-transform, then $n.\hat{x}(n)$ has a rational z-transform whose poles correspond to the poles and zeros of the z-transform of $x(n)$.

[19]. $\hat{x}(0)=\log|x(0)|$.

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